Profesor:
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PITAGORAS
ACADEMIA

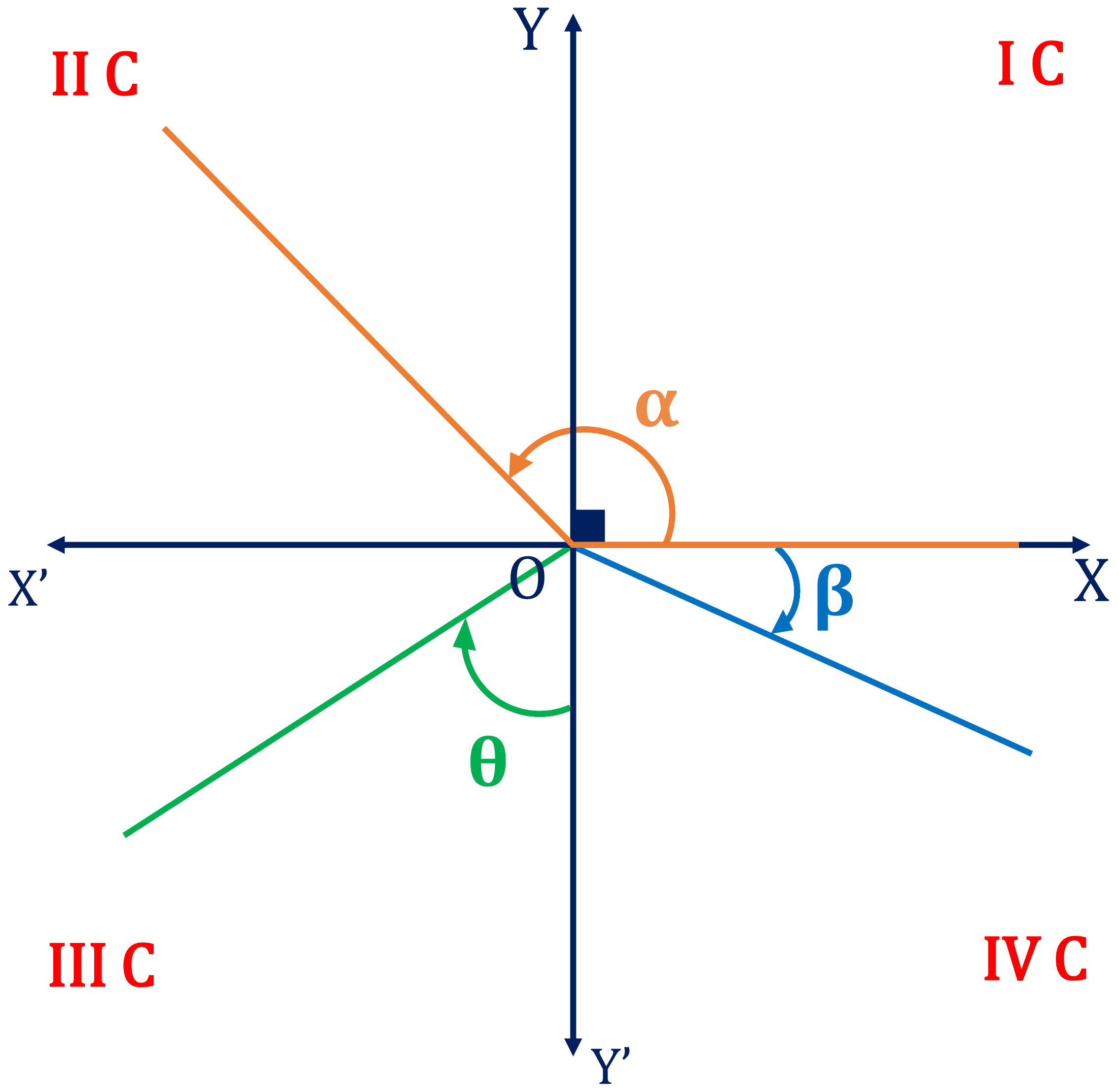
TRIGONOMETRÍA

GRUPO PITÁGORAS

RAZONES TRIGONOMÉTRICAS

RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

1.-Ángulo en posición estándar:



- α es un ángulo en posición estándar

  $\alpha \in IIC$, $m\sphericalangle\alpha > 0$

- β es un ángulo en posición estándar

  $\beta \in IVC$, $m\sphericalangle\beta < 0$

- θ: no es un ángulo en posición estándar

Ejemplos:

Determinar si es verdadero (V) o falso (F) :

(F) Todo ángulo en posición estándar perteneciente al IC es positivo.

(F) La mitad de todo ángulo perteneciente al IC también pertenece al IC.

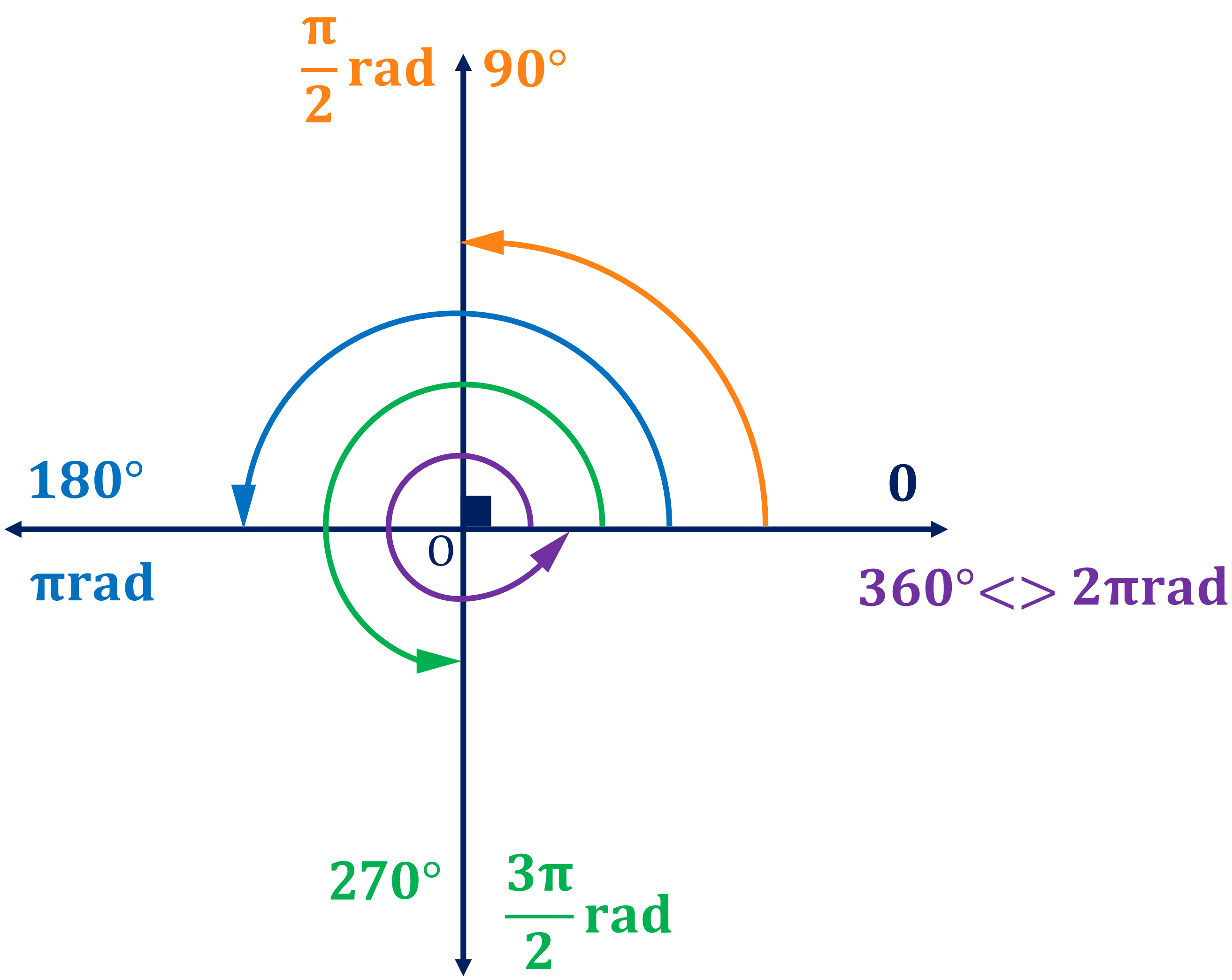
(F) Un ángulo que mide $\frac{5\pi}{2}$ rad pertenece al IIC.

(F) $180° + \alpha$, pertenece al IIIC.

A) FFFF B) VVFF C) VFVF

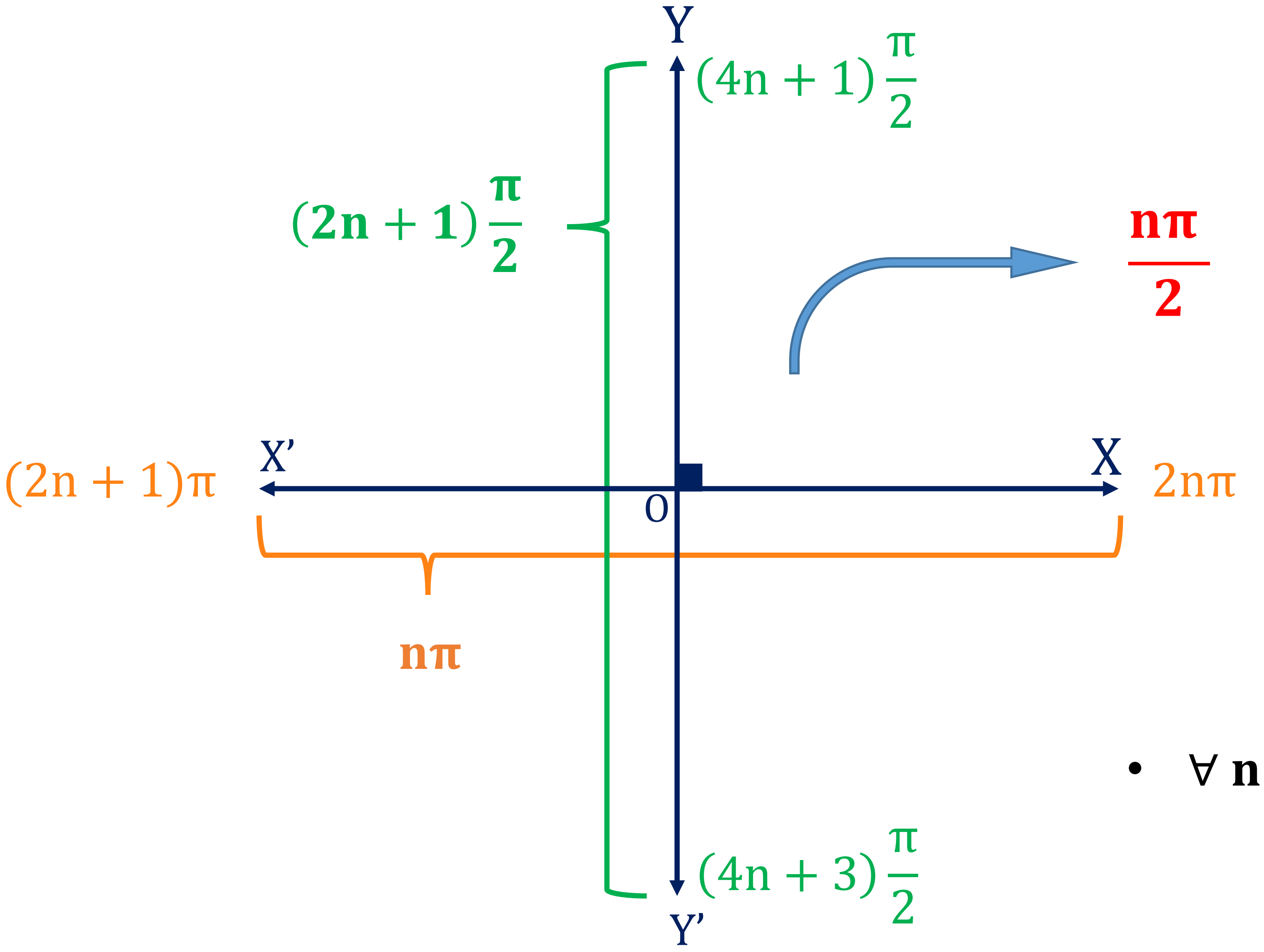
D) FVFV E) FVVV

2.-Ángulo cuadrantal:



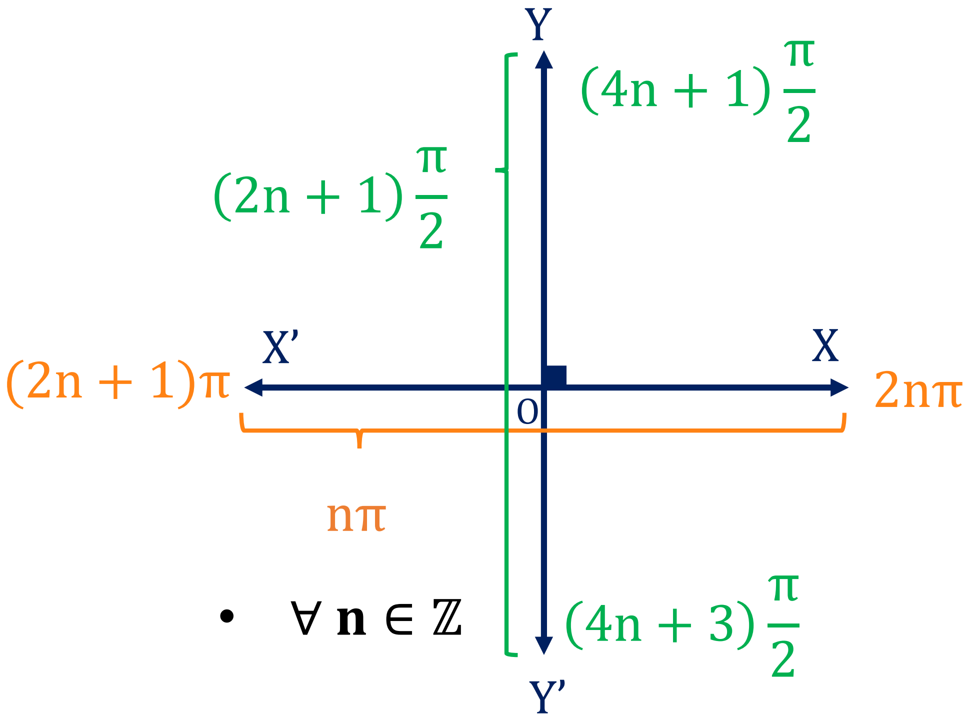
- Múltiplo de $90n°$ $\vee$ $\frac{n\pi}{2}$rad
  - $\forall\ \mathbf{n} \in \mathbb{Z}$

2.1- Forma general de los ángulos cuadrantales:



- $\forall\ \mathbf{n} \in \mathbb{Z}$

RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR



- $\forall\ \mathbf{n} \in \mathbb{Z}$

- $\overrightarrow{OX}: \{2n\pi; \forall\ \mathbf{n} \in \mathbb{Z}\}$
- $\overrightarrow{OX'}: \{(2n+1)\pi; \forall\ \mathbf{n} \in \mathbb{Z}\}$
- $\overrightarrow{OY}: \left\{\frac{(4n+1)\pi}{2}; \forall\ \mathbf{n} \in \mathbb{Z}\right\}$
- $\overrightarrow{OY'}: \left\{\frac{(4n+3)\pi}{2}; \forall\ \mathbf{n} \in \mathbb{Z}\right\}$
- $\overleftrightarrow{X'X}: \{\mathbf{n\pi}; \forall\ \mathbf{n} \in \mathbb{Z}\}$
- $\overleftrightarrow{Y'Y}: \left\{\frac{(2n+1)\pi}{2}; \forall\ \mathbf{n} \in \mathbb{Z}\right\}$

- Si: $\alpha \in IC \leftrightarrow 2n\pi < \alpha < \frac{(4n+1)\pi}{2}$
- Si: $\alpha \in IIC \leftrightarrow \frac{(4n+1)\pi}{2} < \alpha < (2n+1)\pi$
- Si: $\alpha \in IIIC \leftrightarrow (2n+1)\pi < \alpha < \frac{(4n+3)\pi}{2}$
- Si: $\alpha \in IVC \leftrightarrow \frac{(4n+3)\pi}{2} < \alpha < 2(n+1)\pi$

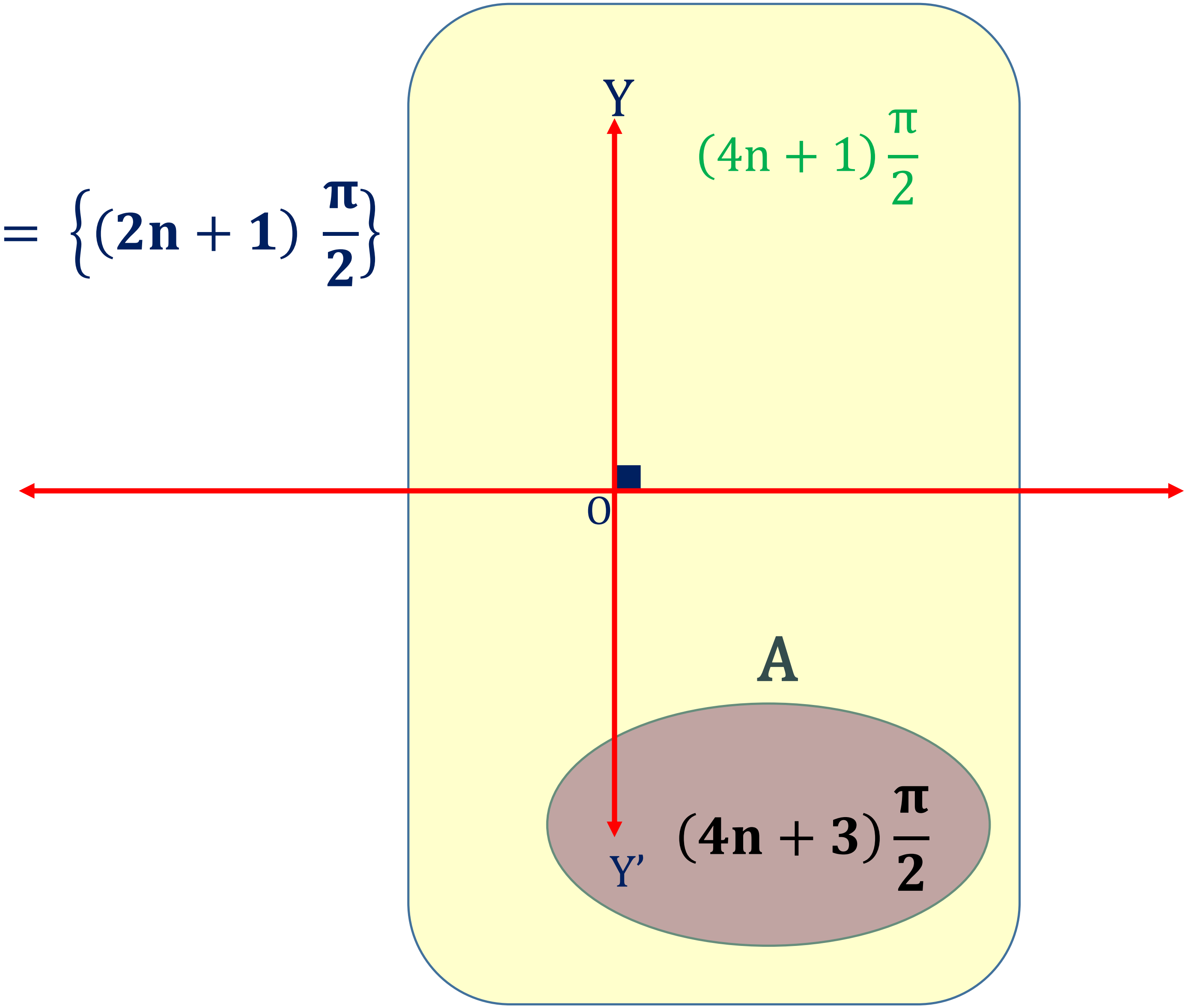
Ejemplos:

Sea A el conjunto de ángulos cuadrantales cuyo lado final pertenece a $\overrightarrow{OY'}$ y B el conjunto de ángulos cuadrantales cuyo lado final pertenece al eje de ordenadas, calcular B-A ($n \in \mathbb{Z}$)

A) $\frac{(4n-1)\pi}{2}$ B) $\frac{(4n+1)\pi}{2}$ C) $\frac{(2n-1)\pi}{2}$ D) $\frac{(2n+1)\pi}{2}$ E) $2n\pi$

RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

$$\mathbf{B} = \left\{(\mathbf{2n+1})\ \frac{\boldsymbol{\pi}}{\mathbf{2}}\right\}$$



$$\mathbf{B} - \mathbf{A} = \left\{(\mathbf{4n+1})\ \frac{\boldsymbol{\pi}}{\mathbf{2}}\right\}$$

Ejemplos:

Sea α un ángulo positivo menor de una vuelta perteneciente al IVC, tal que:

$$\alpha = \left(\frac{\pi}{23} + \frac{2\pi}{23} + \frac{3\pi}{23} + \cdots + \frac{n\pi}{23}\right)\text{rad};\ n \in \mathbb{Z}$$

Calcular la diferencia entre el mayor y menor valor que pueda tomar α.

A) $9\pi/23$ B) $15\pi/23$ C) $13\pi/23$

D) $6\pi/23$ E) $\pi/23$

RESOLUCIÓN:

$\alpha \in IVC \longrightarrow \dfrac{3\pi}{2} < \alpha < 2\pi$

$$\alpha = \left(\frac{\pi}{23} + \frac{2\pi}{23} + \frac{3\pi}{23} + \cdots + \frac{n\pi}{23}\right)\text{rad};\ n \in \mathbb{Z}$$

$$\alpha = \frac{\pi}{23}\,\frac{(n)(n+1)}{2} \quad \ldots\ (i)$$

$$\frac{3\cancel{\pi}}{\cancel{2}} < \frac{\cancel{\pi}}{23} \times \frac{(n)(n+1)}{\cancel{2}} < \frac{4\cancel{\pi}}{\cancel{2}}$$

$$69 < (n)(n+1) < 92$$

$n_{mín} = 8 \longrightarrow 69 < (8)(9) < 92$

$n_{máx} = 9 \longrightarrow 69 < (9)(10) < 92$

- Reemplazando en (i):

$n_{mín} = 8$

$\alpha = \dfrac{\pi}{23}\,\dfrac{(8)(9)}{2} \longrightarrow \alpha_{mín} = \dfrac{36\pi}{23}$

$n_{máx} = 9$

$\alpha = \dfrac{\pi}{23}\,\dfrac{(9)(10)}{2} \longrightarrow \alpha_{máx} = \dfrac{45\pi}{23}$

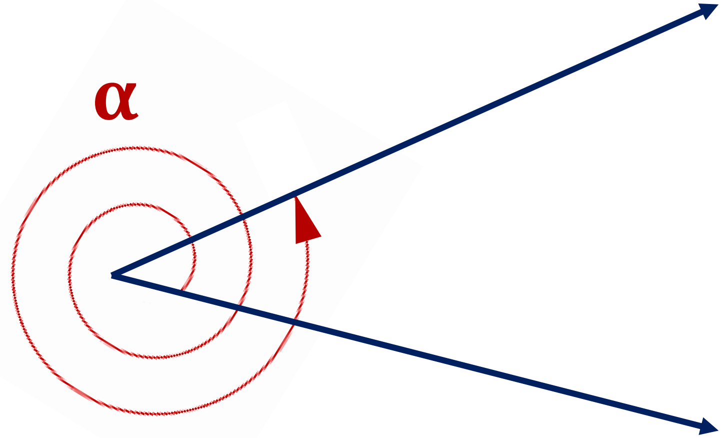
- Calcular la diferencia entre el mayor y menor valor que pueda tomar α.

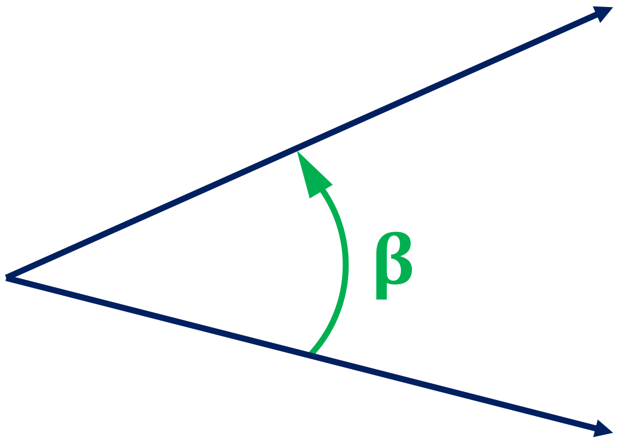
$$\frac{45\pi}{23} - \frac{36\pi}{23}$$

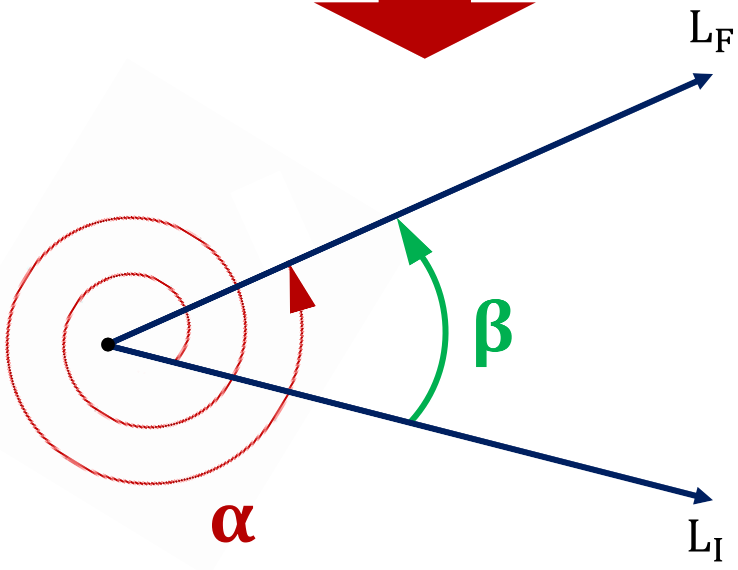
$$\mathbf{\frac{9\pi}{23}}$$

CLAVE: A

3.-Ángulos coterminales:







- $\alpha - \beta = \begin{cases} 360n° \\ 2n\pi rad \end{cases} \forall\, n \in \mathbb{Z}$

- $RT(\alpha) = RT(\beta)$

Ejemplos:

Dos ángulos coterminales están en la relación de 13 a 1, la diferencia de ellos es mayor que 1 200°, pero menor que 1 500°. Hallar el mayor ángulo

A) 1 560° B) 120° C) 1 440° D) 1 000° E) 500°

Resolución:

Sea: α y β los ángulos coterminales $\longrightarrow$ $\alpha - \beta = 360n°$

$\frac{\alpha}{\beta} = \frac{13}{1}$ $\longrightarrow$ $\alpha = 13k \quad \wedge \quad \beta = 1k$

$1200° < \alpha - \beta < 1500°$ $\longrightarrow$ $1200° < 360n° < 1500°$

$360n° = 1440°$

$12k = 1440°$

$\mathbf{k = 120°}$

Hallar el mayor ángulo:

$\alpha = 13(120°)$

$\therefore \boldsymbol{\alpha = 1560°}$

Ejemplos:

Calcular la media aritmética de todos los ángulos positivos coterminales con 230° y menores de 10 000°

A) 5270°   B) 5630°   C) 4910°   D) 5090°   E) 4550°

Resolución:

$$x - 230° = 360n°\,,\ \forall\, n \in \mathbb{Z}$$

$$x = 360n° + 230°$$

$$0 < x < 10000°$$

$$0 < 360n° + 230° < 10000°$$

$$-230° < 360n° < 9770°$$

$$-0{,}63 < n < 27{,}13$$

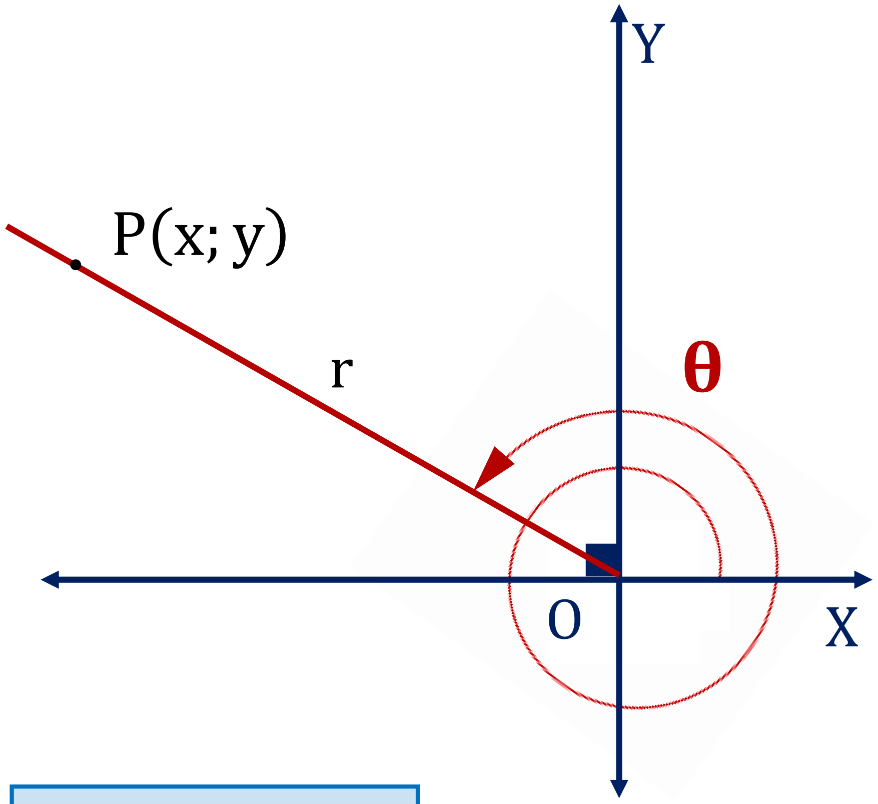
$$n_{\mathbb{Z}} = \underbrace{0, 1, 2\ \ldots 27}_{\text{28 términos}}$$

$$x = 360(0)° + 230°$$
$$x = 360(1)° + 230°$$
$$x = 360(2)° + 230°$$
$$\vdots$$
$$x = 360(27)° + 230°$$

$$MA = \frac{360° \cdot \dfrac{27(27+1)}{2} + 230° \cdot 28}{28}$$

$$\therefore\ \mathbf{MA = 5090°}$$

4.-Cálculo de las razones trigonométricas:



$$\boxed{r = \sqrt{x^2 + y^2}}\ ; r > 0$$

- $x \wedge y \neq 0$

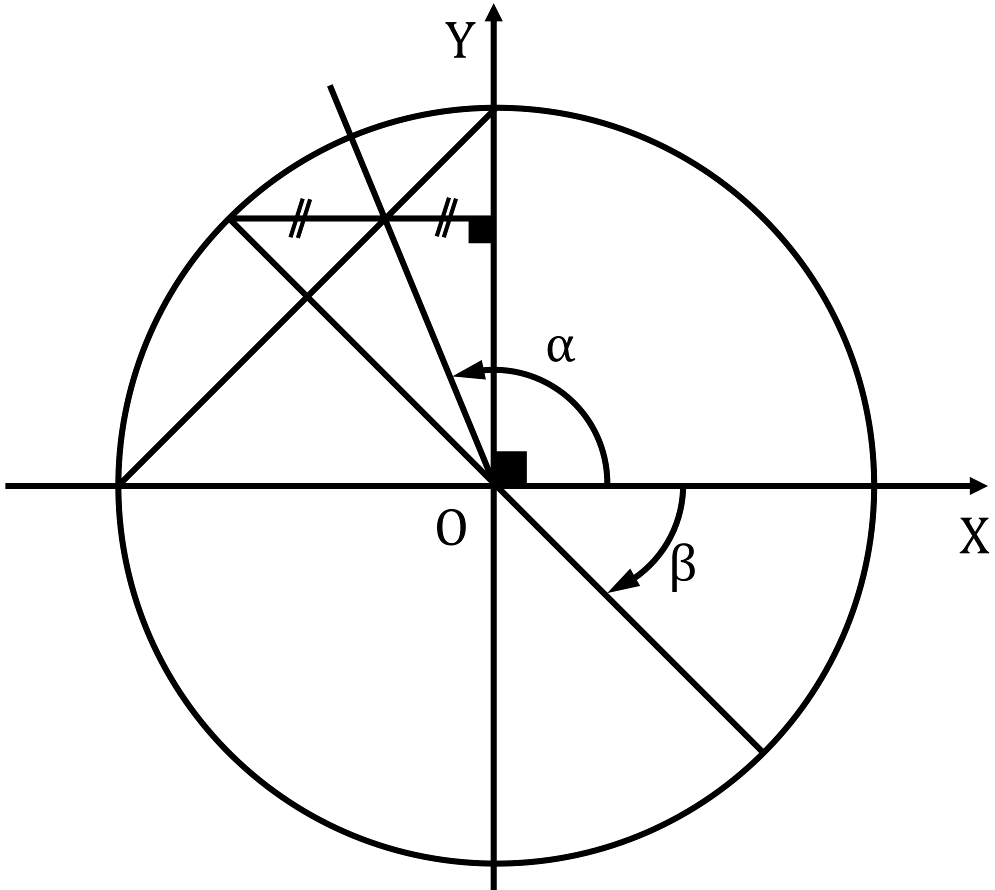
- $\text{Sen}\theta = \dfrac{y}{r}$
- $\text{Csc}\theta = \dfrac{r}{y}$
- $\text{Cos}\theta = \dfrac{x}{r}$
- $\text{Sec}\theta = \dfrac{r}{x}$
- $\text{Tan}\theta = \dfrac{y}{x}$
- $\text{Cot}\theta = \dfrac{x}{y}$

Ejemplos:

Calcular: $\sqrt{13}\text{Cos}\alpha + 10\text{Sen}\beta$

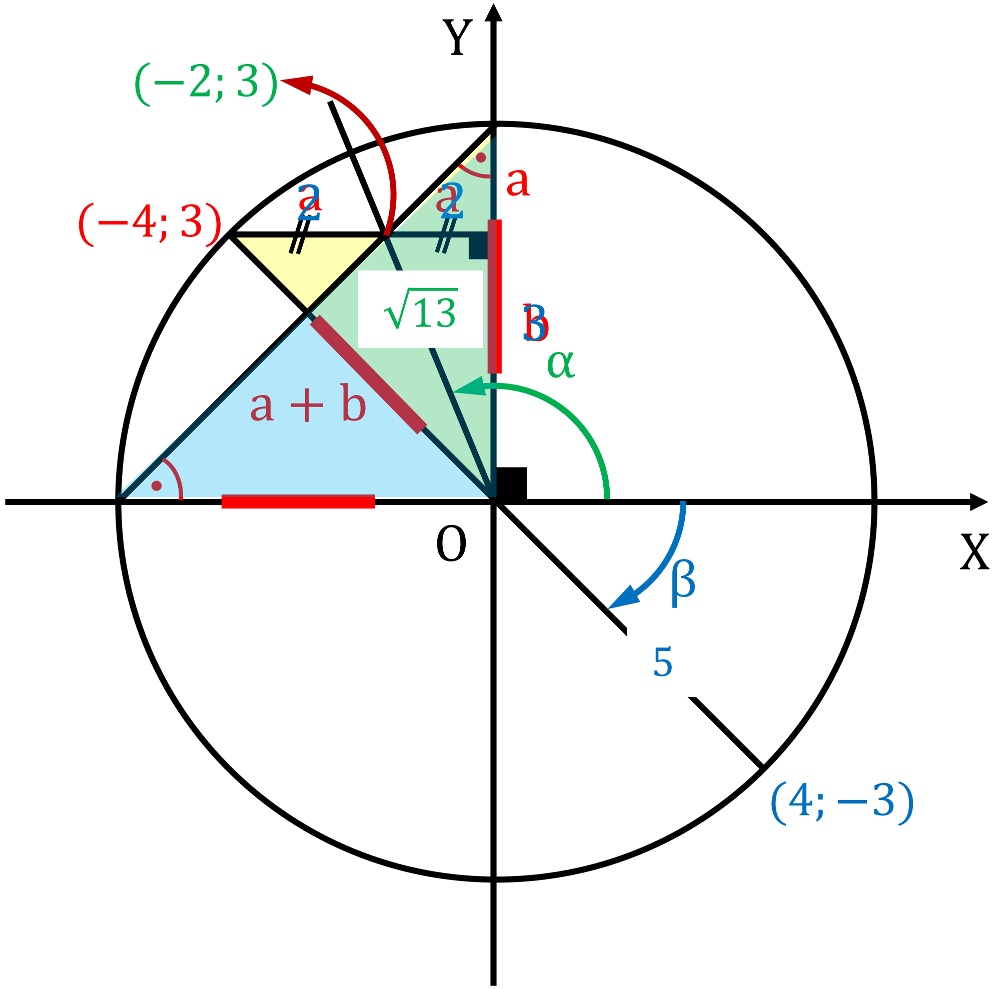
A) -6 B) -8 C) -10

D) -12 E) -14



RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

RESOLUCIÓN:



- Por el Teorema de Pitágoras:

$$(a+b)^2 = (2a)^2 + b^2$$

$$a^2 + 2ab + \cancel{b^2} = 4a^2 + \cancel{b^2}$$

$$2\cancel{a}b = 3\cancel{a}^2$$

$$\frac{2}{3} = \frac{a}{b}$$

- Calcular: $\sqrt{13}\text{Cos}\alpha + 10\text{Sen}\beta$

$$\cancel{\sqrt{13}}\left(\frac{-2}{\cancel{\sqrt{13}}}\right) + \cancel{10}\left(\frac{-3}{\cancel{5}}\right)$$

$$-2 - 6$$

$$\mathbf{-8}$$

CLAVE: B

RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

Ejemplos:

Si $\theta \in$ IIC, además:

$$A = \sqrt{\text{Cos}^2\theta\sqrt{\text{Cos}^2\theta\sqrt{\text{Cos}^2\theta \ldots}}}$$

$$B = \sqrt{\text{Cos}^2\theta + \sqrt{\text{Cos}^2\theta + \sqrt{\text{Cos}^2\theta + \cdots}}}$$

Donde: A+B=9/4

Calcular: $2\sqrt{3}\text{Sen}\theta - 4\text{Cos}\theta$

A) $-\sqrt{3}$ B) $2\sqrt{3}$ C) $3\sqrt{3}$

D) $-3\sqrt{3}$ E) $2\sqrt{3}/3$

RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

RESOLUCIÓN:

- $A = \sqrt{\cos^2\theta\underbrace{\sqrt{\cos^2\theta\sqrt{\cos^2\theta \ldots}}}_{A}}$

$A = \sqrt{\cos^2\theta . A}$

$(A)^2 = \left(\cancel{\sqrt{}}\cos^2\theta . A\right)^{\cancel{2}}$

$A^{\cancel{2}} = \cos^2\theta . \cancel{A}$

$A = \cos^2\theta$

$\dfrac{\mathbf{3}}{\mathbf{4}} = \cos^2\theta \qquad \theta \in \text{IIC}$

$-\dfrac{\sqrt{3}}{\mathbf{2}} = \cos\theta$

- $B = \sqrt{\cos^2\theta + \underbrace{\sqrt{\cos^2\theta + \sqrt{\cos^2\theta + \cdots}}}_{B}}$

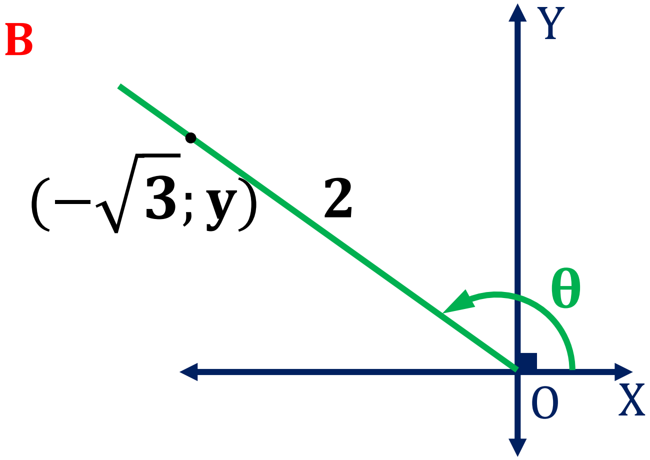
$B = \sqrt{\cos^2\theta + B}$

$(B)^2 = \left(\cancel{\sqrt{}}\cos^2\theta + B\right)^{\cancel{2}}$

$B^2 = \cos^2\theta + B$

$B^2 = A + B$

$B^2 = \dfrac{9}{4} \longrightarrow \mathbf{B} = \dfrac{\mathbf{3}}{\mathbf{2}}$



$(2)^2 = \left(\sqrt{3}\right)^2 + y^2$

$1 = y^2 \longrightarrow 1 = y$

- $\dfrac{9}{4} = A + B$

$\dfrac{9}{4} = A + \dfrac{6}{4} \longrightarrow \dfrac{\mathbf{3}}{\mathbf{4}} = \mathbf{A}$

Calcular: $2\sqrt{3}\text{Sen}\theta - 4\text{Cos}\theta$

$\cancel{2}\sqrt{3}\left(\dfrac{1}{\cancel{2}}\right) - \cancel{4}\left(\dfrac{-\sqrt{3}}{\cancel{2}}\right)$

$\mathbf{3\sqrt{3}}$

CLAVE: C

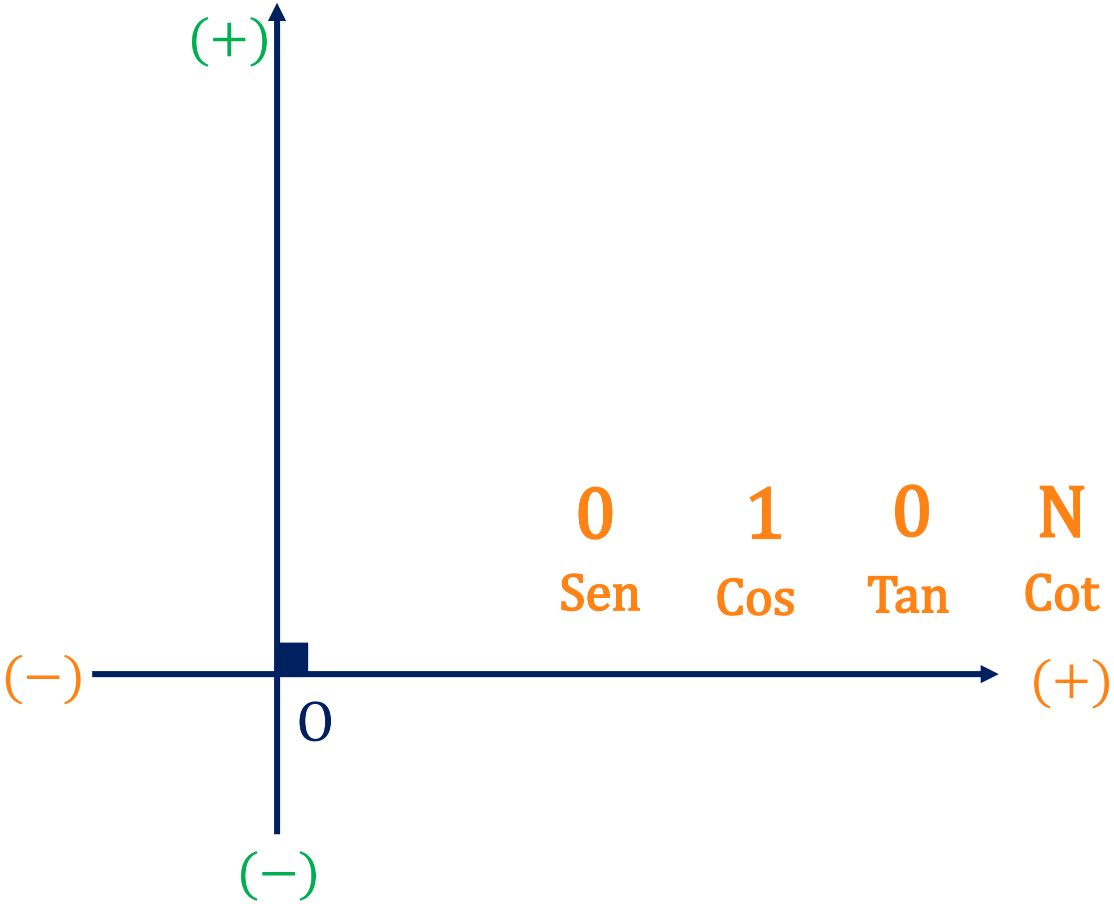
RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

5.-Razones trigonométricas de los ángulos cuadrantales:

Sen	Cos	Tan	Cot	Sec	Csc
1	0	N	0	N	1

- N: no definido

Sen	Cos	Tan	Cot	Sec	Csc
0	1	0	N	1	N



Ejemplos:

Sean α y β ángulos coterminales que están en la relación de 7 a 29, además la suma de ellos se encuentra en el intervalo]720°; 1440°[. Hallar el seno del ángulo mayor multiplicado por $\frac{22}{36}$ sumado del ángulo menor multiplicado por $\frac{11}{18}$.

A) -1 B) $\sqrt{3}/5$ C) $(\sqrt{2}+\sqrt{3})/7$

D) 0 E) $-3/5$

RESOLUCIÓN:

- $\alpha \wedge \beta$: $\sphericalangle_s$ coterminales

$$\frac{\beta}{\alpha} = \frac{7k}{29k} \longrightarrow \beta = 7k\,,\ \alpha = 29k$$

i. $720° < \alpha + \beta < 1440°$

$$720° < 36k < 1440°$$

$$20° < k < 40° \xrightarrow{x11} 220° < 11k < 440°$$

ii. $\alpha - \beta = 360n°$

$$22k = 360n°$$

$$11k = 180n°$$

$$11k = 360°$$

- $\text{Sen}\left(29k.\frac{22}{36} + 7k.\frac{22}{36}\right)$

$$\text{Sen}\left[\frac{22}{\cancel{36}}\cancel{(36k)}\right]$$

$$\text{Sen}(22k)$$

$$\text{Sen}(720°)$$

0

CLAVE: D

Ejemplos:

Si se cumplen las siguientes condiciones: $\sqrt{\text{Cos}^3\alpha + \text{Cos}^2\alpha - \text{Cos}\alpha - 1} = \text{Tan}\beta$, $|\text{Sen}\theta - 1| = \text{Sen}\theta$, halle la suma de los valores que toma la siguiente expresión:

$$\text{Cos}\beta + \text{Cos}\alpha + \text{Sen}\theta$$

A) -1 B) $3/2$ C) $-3/2$ D) $1/2$ E) 2

RESOLUCIÓN:

- $\sqrt{\mathrm{Cos}^3\alpha + \mathrm{Cos}^2\alpha - \mathrm{Cos}\alpha - 1} = \mathrm{Tan}\beta$

$$\sqrt{\mathrm{Cos}^2\alpha(\mathrm{Cos}\alpha + 1) - (\mathrm{Cos}\alpha + 1)} = \mathrm{Tan}\beta$$

$$\sqrt{(\mathrm{Cos}\alpha + 1)(\mathrm{Cos}^2\alpha - 1)} = \mathrm{Tan}\beta$$

$$\sqrt{\underbrace{(\mathrm{Cos}\alpha + 1)}_{"+"\ \vee\ "0"}\underbrace{\underbrace{(-\mathrm{Sen}^2\alpha)}_{"-"\ \vee\ "0"}}_{0}} = \mathrm{Tan}\beta$$

$$\boxed{\mathrm{Cos}\alpha = \pm 1} \qquad \boxed{\mathrm{Tan}\beta = 0}$$

- $|\mathrm{Sen}\theta - 1| = \mathrm{Sen}\theta$

$$\mathrm{Sen}\theta - 1 = \mathrm{Sen}\theta \quad \vee \quad \mathrm{Sen}\theta - 1 = -\mathrm{Sen}\theta$$

$$-1 = 0 \qquad\qquad 2\mathrm{Sen}\theta = 1$$

$$\boxed{\mathrm{Sen}\theta = \frac{1}{2}}$$

- $\mathrm{Cos}\beta + \mathrm{Cos}\alpha + \mathrm{Sen}\theta$

i. $(1) + (1) + \frac{1}{2} = \frac{5}{2}$

ii. $(1) + (-1) + \frac{1}{2} = \frac{1}{2}$

iii. $(-1) + (1) + \frac{1}{2} = \frac{1}{2}$

iv. $(-1) + (-1) + \frac{1}{2} = -\frac{3}{2}$

$$\sum \mathrm{Valores} = \mathbf{2}$$

CLAVE: E

RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

6.-Signos de las razones trigonométricas:

Sen Csc (+)	Todas (+)
Tan Cot (+)	Cos Sec (+)

Ejemplos:

Determinar el signo de P;Q y R:

$$P = \mathrm{Sen}\left(\frac{1997\pi}{7}\right)$$

$$Q = \mathrm{Tan}\left(\frac{1471\pi}{6}\right)$$

$$R = \mathrm{Cos}\left(\frac{3852\pi}{5}\right)$$

A) $+;+;+$ B) $-;-;-$ C) $+;+;-$

D) $-;+;+$ E) $-;-;+$

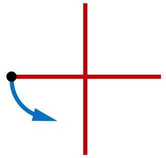
RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

RESOLUCIÓN:

$$P = \operatorname{Sen}\left(\frac{1997\pi}{7}\right)$$

$$\begin{array}{r|l} 1997\pi & 7 \\ \cline{2-2} 1995\pi & 285\pi \\ \hline 2\pi & \end{array}$$

$$P = \operatorname{Sen}\left(285\pi + \frac{2\pi}{7}\right)$$

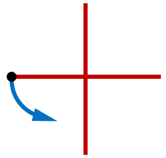


$$P = (-)$$

$$Q = \operatorname{Tan}\left(\frac{1471\pi}{6}\right)$$

$$\begin{array}{r|l} 1471\pi & 6 \\ \cline{2-2} 1470\pi & 245\pi \\ \hline 1\pi & \end{array}$$

$$Q = \operatorname{Tan}\left(245\pi + \frac{\pi}{6}\right)$$

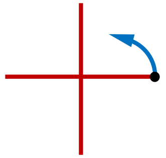


$$Q = (+)$$

$$R = \operatorname{Cos}\left(\frac{3852\pi}{5}\right)$$

$$\begin{array}{r|l} 3852\pi & 5 \\ \cline{2-2} 3850\pi & 770\pi \\ \hline 2\pi & \end{array}$$

$$R = \operatorname{Cos}\left(770\pi + \frac{2\pi}{5}\right)$$



$$R = (+)$$

CLAVE: D

Ejemplos:

Si α y β ángulos en posición estándar positivos, menores de una vuelta, ubicados en cuadrantes diferentes y consecutivos tal que $\alpha > \beta$, si se cumplen:

$\text{Sen}\alpha . \text{Cos}\beta > 0$ y

$\text{Tan}\alpha . \text{Sec}\beta > 0$

Determinar el signo en cada caso:

I. $\text{Cos}\beta + \text{Sen}\alpha$

II. $\text{Cot}^3\beta - \text{Cot}^3\alpha$

III. $\text{Csc}^4\beta . \text{Csc}\alpha$

A) $-; +; -$ B) $-; -; +$ C) $+; +; -$

D) $+; +; +$ E) $-; -; -$

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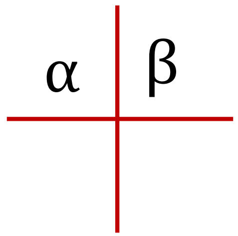
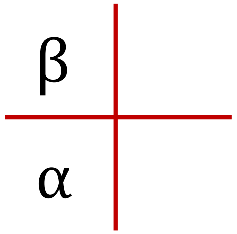
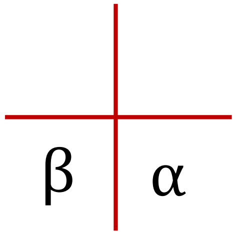
RT DE UN ÁNGULO EN POSICIÓN ESTÁNDAR

RESOLUCIÓN:

$\alpha > \beta$	Dato I		Dato II	
	$(+)(+)$	$(-)(-)$	$(+)(+)$	$(-)(-)$
i. 	✓	✕	✕	✕
ii. 	✕	✓	✕	✕
iii. 	✕	✓	✕	✓

- $\text{Sen}\alpha.\text{Cos}\beta > 0$
  $(+)\quad(+)$
  $(-)\quad(-)$
- $\text{Tan}\alpha.\text{Sec}\beta > 0$
  $(+)\quad(+)$
  $(-)\quad(-)$

$\longrightarrow$ $\alpha \in \text{IVC} \wedge \ \beta \in \text{IIIC}$

I. $\text{Cos}\beta + \text{Sen}\alpha$

$(-) + (-) = (-)$

II. $\text{Cot}^3\beta - \text{Cot}^3\alpha$

$(+) - (-) = (+)$

III. $\text{Csc}^4\beta.\text{Csc}\alpha$

$(+) . (-) = (-)$

CLAVE: A

RAZONES TRIGONOMÉTRICAS

REDUCCIÓN AL PRIMER CUADRANTE

REDUCCIÓN AL PRIMER CUADRANTE

1erCASO: Para ángulos positivos y menores a una vuelta

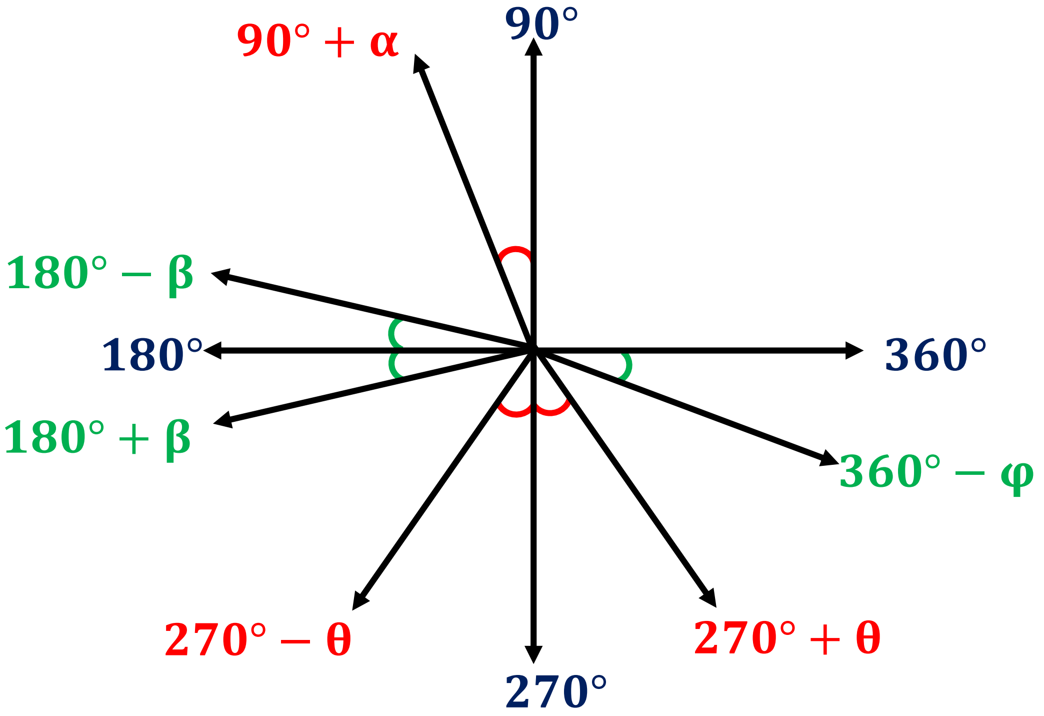
$$RT(90° + \alpha) = \pm \ \text{CO-RT}(\alpha)$$

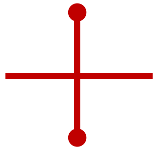
$$RT(180° \pm \beta) = \pm \ RT(\beta)$$

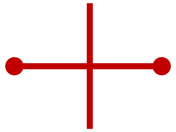
$$RT(270° \pm \theta) = \pm \ \text{CO-RT}(\theta)$$

$$RT(360° - \varphi) = \pm \ RT(\varphi)$$

1er paso	2do paso	3er paso
Descomponer	**Signo**	**¿ Cambia?**



 **SI**

 **NO**

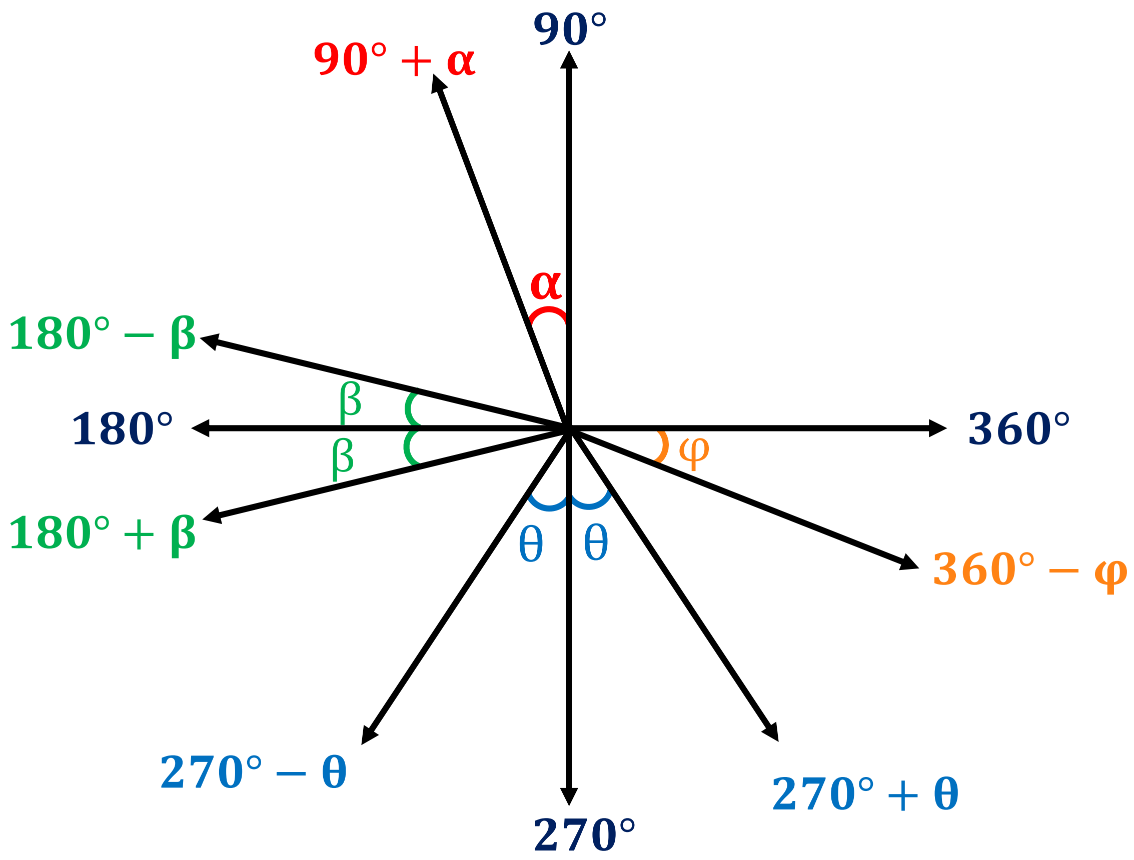
REDUCCIÓN AL PRIMER CUADRANTE

❖ Nota:

a) El signo $(\pm)$ dependerá del cuadrante y la RT del ángulo a reducir.

b) Para ubicar el cuadrante consideremos el ángulo a trabajar como (agudo), $RT\left(\frac{n\pi}{2} \pm \theta\right)\ 0 < \theta < 90°$

c) Gráficamente para un ángulo agudo.



REDUCCIÓN AL PRIMER CUADRANTE

Calcular : $\text{Sen}135°\text{Csc}225° + \text{Tan}240°\text{Cot}330°$

A) 4 ~~B)~~ -4 C) 2 D) -2 E) 0

Resolución:

$$\underbrace{\underbrace{\text{Sen}(180° - 45°)}_{(+\text{Sen}45°)} \times \underbrace{\text{Csc}(180° + 45°)}_{(-\text{Csc}45°)}}_{-1} + \underbrace{\underbrace{\text{Tan}(180° + 60°)}_{(+\text{Tan}60°)} \times \underbrace{\text{Cot}(270° + 60°)}_{(-\text{Tan}60°)}}_{-3}$$

$$-1 + -3$$

-4

REDUCCIÓN AL PRIMER CUADRANTE

Calcular:
$$E = \frac{\operatorname{Sec}\left(\frac{4\pi}{3}\right)\operatorname{Csc}\left(\frac{7\pi}{6}\right)\operatorname{Tan}\left(\frac{2\pi}{3}\right)}{\operatorname{Sen}\left(\frac{3\pi}{4}\right)\operatorname{Cos}\left(\frac{5\pi}{3}\right)\operatorname{Tan}\left(\frac{5\pi}{6}\right)}$$

A) $12\sqrt{2}$ B) $24\sqrt{2}$ C) $-12\sqrt{2}$ D) $-24\sqrt{2}$ E) $48\sqrt{2}$

Resolución:

$$E = \frac{\operatorname{Sec}\left(\pi + \frac{\pi}{3}\right)\operatorname{Csc}\left(\pi + \frac{\pi}{6}\right)\operatorname{Tan}\left(\pi - \frac{\pi}{3}\right)}{\operatorname{Sen}\left(\pi - \frac{\pi}{4}\right)\operatorname{Cos}\left(2\pi - \frac{\pi}{3}\right)\operatorname{Tan}\left(\pi - \frac{\pi}{6}\right)}$$

$$E = \frac{\left[\cancel{-}\operatorname{Sec}\left(\frac{\pi}{3}\right)\right]\left[\cancel{-}\operatorname{Csc}\left(\frac{\pi}{6}\right)\right]\left[\cancel{-}\operatorname{Tan}\left(\frac{\pi}{3}\right)\right]}{\left[+\operatorname{Sen}\left(\frac{\pi}{4}\right)\right]\left[+\operatorname{Cos}\left(\frac{\pi}{3}\right)\right]\left[\cancel{-}\operatorname{Tan}\left(\frac{\pi}{6}\right)\right]}$$

$$E = \frac{(2)(2)(\sqrt{3})}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$\therefore \mathbf{E = 24\sqrt{2}}$$

REDUCCIÓN AL PRIMER CUADRANTE

❖ Ángulos relacionados:

i. Ángulos complementarios:

$$\boxed{x + y = 90°}$$

$$\mathrm{Senx} = \mathrm{Cosy}$$

$$\mathrm{Tanx} = \mathrm{Coty}$$

$$\mathrm{Secx} = \mathrm{Cscy}$$

ii. Ángulos suplementarios:

$$\boxed{x + y = 180°}$$

$$\mathrm{Senx} = \mathrm{Seny}$$

$$\mathrm{Cosx} = -\mathrm{Cosy}$$

$$\mathrm{Tanx} = -\mathrm{Tany}$$

iii. Ángulos explementarios:

$$\boxed{x + y = 360°}$$

$$\mathrm{Senx} = -\mathrm{Seny}$$

$$\mathrm{Cosx} = \mathrm{Cosy}$$

$$\mathrm{Tanx} = -\mathrm{Tany}$$

REDUCCIÓN AL PRIMER CUADRANTE

Sabiendo que $A + B = 180°$,calcular : $E = \dfrac{\text{Cos}(A)\text{Cot}\left(\frac{B}{2}\right)}{\text{Tan}\left(\frac{A}{2}\right)\text{Cos}(B)}$

A) 1 B) -1 C) 0 D) 2 E) -2

Resolución:

$A + B = 180°$ ⟹ **Suplementarios** $\left[\ \text{CosA} = -\text{CosB}\right.$

$\dfrac{A}{2} + \dfrac{B}{2} = 90°$ ⟹ **Complementarios** $\left[\ \text{Tan}\left(\dfrac{A}{2}\right) = \text{Cot}\left(\dfrac{B}{2}\right)\right.$

$$E = \frac{(-\text{CosB})\text{Cot}\left(\frac{B}{2}\right)}{\text{Cot}\left(\frac{B}{2}\right)\text{CosB}}$$

$\therefore \mathbf{E = -1}$

REDUCCIÓN AL PRIMER CUADRANTE

Calcular el valor de: $\text{Sen}^{2n+1}\frac{5\pi}{17} + \text{Sen}^{2n+1}\frac{11\pi}{17} + \text{Sen}^{2n+1}\frac{23\pi}{17} + \text{Sen}^{2n+1}\frac{29\pi}{17}$

A) -1 B) 1 C) 1/2 D) -2 E) 0

Resolución:

$+2\pi$

$$\underbrace{\cancel{\text{Sen}^{2n+1}\frac{5\pi}{17}} + \cancel{\text{Sen}^{2n+1}\frac{11\pi}{17}} + \text{Sen}^{2n+1}\frac{23\pi}{17} + \text{Sen}^{2n+1}\frac{29\pi}{17}}_{0}$$

$$\cancel{\left(-\text{Sen}\frac{11\pi}{17}\right)}^{2n+1} \qquad \cancel{\left(-\text{Sen}\frac{5\pi}{17}\right)}^{2n+1}$$

$\measuredangle_s$ Explementarios:

Si: $x + y = 2\pi$

$\text{Sen}x = -\text{Sen}y$

2^{do} CASO: Para ángulos positivos y mayores a una vuelta

Sea $\beta > 360°$:

$$\boxed{\mathbf{RT(\beta) = RT(\alpha)}}$$

Donde α es el residuo de:

$$\begin{array}{l|l} \beta & 360° \\ \cline{2-2} & \quad q \\ \alpha & \end{array}$$

❖ OBSERVACIÓN:

$$\boxed{\mathbf{RT(\cancel{2n\pi} + \beta) = RT(\beta)}}$$

REDUCCIÓN AL PRIMER CUADRANTE

❖ Conclusión:

- ✓ $\mathbf{RT\{n\pi \pm \alpha\} = \pm RT(\alpha)}$
- ✓ $\mathbf{RT\left\{(2n+1)\frac{\pi}{2} \pm \beta\right\} = \pm CO-RT(\beta)}$

REDUCCIÓN AL PRIMER CUADRANTE

Calcular : $\dfrac{\text{Cos}3660°}{\text{Sen}840°}$

A) $\sqrt{3}$ B) $-\sqrt{3}$ C) $-\frac{\sqrt{3}}{3}$ D) $\frac{\sqrt{3}}{3}$ E) 1

Resolución:

$$3660° = 360° \cdot 10 + 60°$$

$$840° = 360° \cdot 2 + 120°$$

$$\frac{\text{Cos}(60°)}{\text{Sen}(120°)} = \frac{\text{Cos}60°}{\text{Sen}(180° - 60°)} = \frac{\text{Cos}60°}{\text{Sen}60°} = \text{Cot}60° = \frac{\sqrt{3}}{3}$$

REDUCCIÓN AL PRIMER CUADRANTE

Calcular el valor de la siguiente expresión: $M = \text{Sen}\left(\frac{245\pi}{6}\right)\text{Cos}\left(\frac{163\pi}{4}\right)\text{Tan}\left(\frac{77\pi}{3}\right)$

A) $\frac{\sqrt{6}}{2}$ B) $\frac{\sqrt{6}}{3}$ C) $\frac{\sqrt{6}}{4}$ D) $\frac{\sqrt{6}}{5}$ E) $\frac{\sqrt{6}}{6}$

Resolución:

$$\begin{array}{r|l} 245 & 6 \\ \hline -1 & 41 \end{array} \qquad \begin{array}{r|l} 163 & 4 \\ \hline -1 & 41 \end{array} \qquad \begin{array}{r|l} 77 & 3 \\ \hline -1 & 26 \end{array}$$

$$M = \text{Sen}\left(41\pi - \frac{1\pi}{6}\right)\text{Cos}\left(41\pi - \frac{1\pi}{4}\right)\text{Tan}\left(26\pi - \frac{1\pi}{3}\right)$$

$$M = \left[+\text{Sen}\left(\frac{\pi}{6}\right)\right]\left[-\text{Cos}\left(\frac{\pi}{4}\right)\right]\left[-\text{Tan}\left(\frac{\pi}{3}\right)\right]$$

$$M = \left[\left(\frac{1}{2}\right)\right]\left[-\left(\frac{\sqrt{2}}{2}\right)\right]\left[-(\sqrt{3})\right]$$

$$\therefore\ \mathbf{M = \frac{\sqrt{6}}{4}}$$

REDUCCIÓN AL PRIMER CUADRANTE

3^{er} CASO: Para ángulos negativos

Sea $x > 0$

$$\mathrm{Sen}(-x) = -\mathrm{Senx}$$

$$\mathrm{Cos}(-x) = \mathrm{Cosx}$$

$$\mathrm{Tan}(-x) = -\mathrm{Tanx}$$

$$\mathrm{Cot}(-x) = -\mathrm{Cotx}$$

$$\mathrm{Sec}(-x) = \mathrm{Secx}$$

$$\mathrm{Csc}(-x) = -\mathrm{Cscx}$$

REDUCCIÓN AL PRIMER CUADRANTE

Simplificar : $E = \dfrac{\operatorname{Sen}(-120°) - \operatorname{Cos}(-210°) + \operatorname{Sec}(-300°)}{\operatorname{Tan}(-135°) + \operatorname{Sec}(-225°) + \operatorname{Sec}(-315°)}$

A) -2 B) -3 C) 1 D) -1 E) 2

Resolución:

$$E = \frac{[-\operatorname{Sen}(120°)] - [\operatorname{Cos}(210°)] + [\operatorname{Sec}(300°)]}{[-\operatorname{Tan}(135°)] + [\operatorname{Sec}(225°)] + [\operatorname{Sec}(315°)]}$$

$$E = \frac{[-\operatorname{Sen}(180° - 60°)] - [\operatorname{Cos}(270° - 60°)] + [\operatorname{Sec}(360° - 60°)]}{[-\operatorname{Tan}(180° - 45°)] + [\operatorname{Sec}(180° + 45°)] + [\operatorname{Sec}(360° - 45°)]}$$

$$E = \frac{\cancel{[-(+\operatorname{Sen}60°)]} - \cancel{[-\operatorname{Sen}60°]} + [+\operatorname{Sec}60°]}{[-(-\operatorname{Tan}45°)] + \cancel{[-\operatorname{Sec}45°]} + \cancel{[+\operatorname{Sec}45°]}}$$

$$E = \frac{2}{1}$$

$$\therefore \mathbf{E = 2}$$

Hallar: $B = \dfrac{\text{Sec}(\alpha - 85\pi)\text{Csc}\left(\alpha - \dfrac{39\pi}{2}\right)}{\text{Tan}\left(\alpha - \dfrac{73\pi}{2}\right)\text{Cos}(\alpha - 73\pi)}$; para $\alpha = \dfrac{\pi}{3}$

A) -2 B) $-4\sqrt{3}$ C) 6 D) $-8\sqrt{3}$ E) 2

Resolución:

$$B = \frac{\text{Sec}(85\pi - \alpha)\left[-\text{Csc}\left(\frac{39\pi}{2} - \alpha\right)\right]}{\left[-\text{Tan}\left(\frac{73\pi}{2} - \alpha\right)\right]\text{Cos}(73\pi - \alpha)}$$

$$B = \frac{(-\text{Sec}\alpha)[-(-\text{Sec}\alpha)]}{[-(+\text{Cot}\alpha)](-\text{Cos}\alpha)}$$

$$B = \frac{\left(-\text{Sec}\frac{\pi}{3}\right)\left(\text{Sec}\frac{\pi}{3}\right)}{\left(-\text{Cot}\frac{\pi}{3}\right)\left(-\text{Cos}\frac{\pi}{3}\right)}$$

$$B = -\frac{(2)(2)}{\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{2}\right)}$$

$$\therefore \mathbf{B = -8\sqrt{3}}$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

Problema 1:

Si: $2^{2\text{Tan}\theta+2} - 6^{\text{Tan}\theta} = 2.3^{2\text{Tan}\theta+2}$

Evaluar la expresión : $\text{M} = \text{Sec}\theta + \text{Csc}\theta$, si y solo si: $\frac{\pi}{2} < \theta < \pi$

A) $2\sqrt{5}$ B) $\sqrt{5}$ C) $-\sqrt{5}$

D) $-\sqrt{5}/2$ E) $-2\sqrt{5}$

1

$$2^{2\tan\theta+2}-6^{\tan\theta}=2\cdot3^{2\tan\theta+2} \; ; \; \frac{\pi}{2}<\theta<\pi$$

$$2^{2\tan\theta}\cdot2^{2}-(2\cdot3)^{\tan\theta}=2\cdot3^{2\tan\theta}\cdot3^{2}$$

①

$$2^{2\tan\theta+2} - 6^{\tan\theta} = 2.3^{2\tan\theta+2} \quad ; \quad \frac{\pi}{2} < \theta < \pi$$

$$2^{2\tan\theta} \cdot 2^{2} - (2.3)^{\tan\theta} = 2.3^{2\tan\theta} \cdot 3^{2}$$

$$4.(\underbrace{2^{\tan\theta}}_{x})^{2} - \underbrace{2^{\tan\theta}}_{x} \cdot \underbrace{3^{\tan\theta}}_{y} = 18.(\underbrace{3^{\tan\theta}}_{y})^{2}$$

1

$$2^{2\tan\theta+2}-6^{\tan\theta}=2\cdot 3^{2\tan\theta+2} \quad ; \quad \frac{\pi}{2}<\theta<\pi$$

$$2^{2\tan\theta}\cdot 2^{2}-(2\cdot 3)^{\tan\theta}=2\cdot 3^{2\tan\theta}\cdot 3^{2}$$

$$4\cdot(\underbrace{2^{\tan\theta}}_{x})^{2}-\underbrace{2^{\tan\theta}}_{x}\cdot\underbrace{3^{\tan\theta}}_{y}=18\cdot(\underbrace{3^{\tan\theta}}_{y})^{2}$$

$$4x^{2}-xy-18y^{2}=0$$

1

$$2^{2\tan\theta+2} - 6^{\tan\theta} = 2\cdot 3^{2\tan\theta+2} \quad ; \quad \frac{\pi}{2}<\theta<\pi$$

$$2^{2\tan\theta}\cdot 2^{2} - (2\cdot 3)^{\tan\theta} = 2\cdot 3^{2\tan\theta}\cdot 3^{2}$$

$$4\cdot(\underbrace{2^{\tan\theta}}_{x})^{2} - \underbrace{2^{\tan\theta}}_{x}\cdot\underbrace{3^{\tan\theta}}_{y} = 18\cdot(\underbrace{3^{\tan\theta}}_{y})^{2}$$

$$4x^{2} - xy - 18y^{2} = 0$$

$$\begin{array}{ll} 4x & -9y \\ x & +2y \end{array}$$

$$2^{2\tan\theta+2} - 6^{\tan\theta} = 2.3^{2\tan\theta+2} \quad ; \quad \frac{\pi}{2}<\theta<\pi$$

$$2^{2\tan\theta}.2^{2} - (2.3)^{\tan\theta} = 2.3^{2\tan\theta}.3^{2}$$

$$4.(\underbrace{2^{\tan\theta}}_{x})^{2} - \underbrace{2^{\tan\theta}}_{x}.\underbrace{3^{\tan\theta}}_{y} = 18.(\underbrace{3^{\tan\theta}}_{y})^{2}$$

$$4x^{2} - xy - 18y^{2} = 0$$

$$\begin{array}{cc} 4x & -9y \\ x & +2y \end{array}$$

$$4x = 9y \quad \checkmark \quad x = -2y$$

$$2^{2}.2^{\tan\theta} = 3^{2}.3^{\tan\theta}$$

$$2^{2+\tan\theta} = 3^{2+\tan\theta}$$

$$2+\tan\theta = 0$$

$$\tan\theta = -2$$

$$2^{2\tan\theta+2} - 6^{\tan\theta} = 2\cdot 3^{2\tan\theta+2} \quad ; \quad \frac{\pi}{2}<\theta<\pi$$

$$2^{2\tan\theta}\cdot 2^{2} - (2\cdot 3)^{\tan\theta} = 2\cdot 3^{2\tan\theta}\cdot 3^{2}$$

$$4\cdot(\underbrace{2^{\tan\theta}}_{x})^{2} - \underbrace{2^{\tan\theta}}_{x}\cdot\underbrace{3^{\tan\theta}}_{y} = 18\cdot(\underbrace{3^{\tan\theta}}_{y})^{2}$$

$$4x^{2} - xy - 18y^{2} = 0$$

$$\begin{array}{ll} 4x & -9y \\ x & +2y \end{array}$$

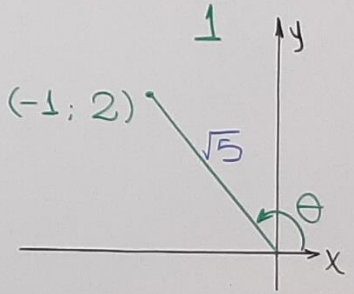
$$\Rightarrow 4x = 9y \quad \vee \quad x = -2y$$

$$2^{2}\cdot 2^{\tan\theta} = 3^{2}\cdot 3^{\tan\theta}$$

$$2^{2+\tan\theta} = 3^{2+\tan\theta}$$

$$2+\tan\theta = 0$$

$$\tan\theta = -\frac{2}{1}$$



Calcular:

$$M = \sec\theta + \csc\theta$$

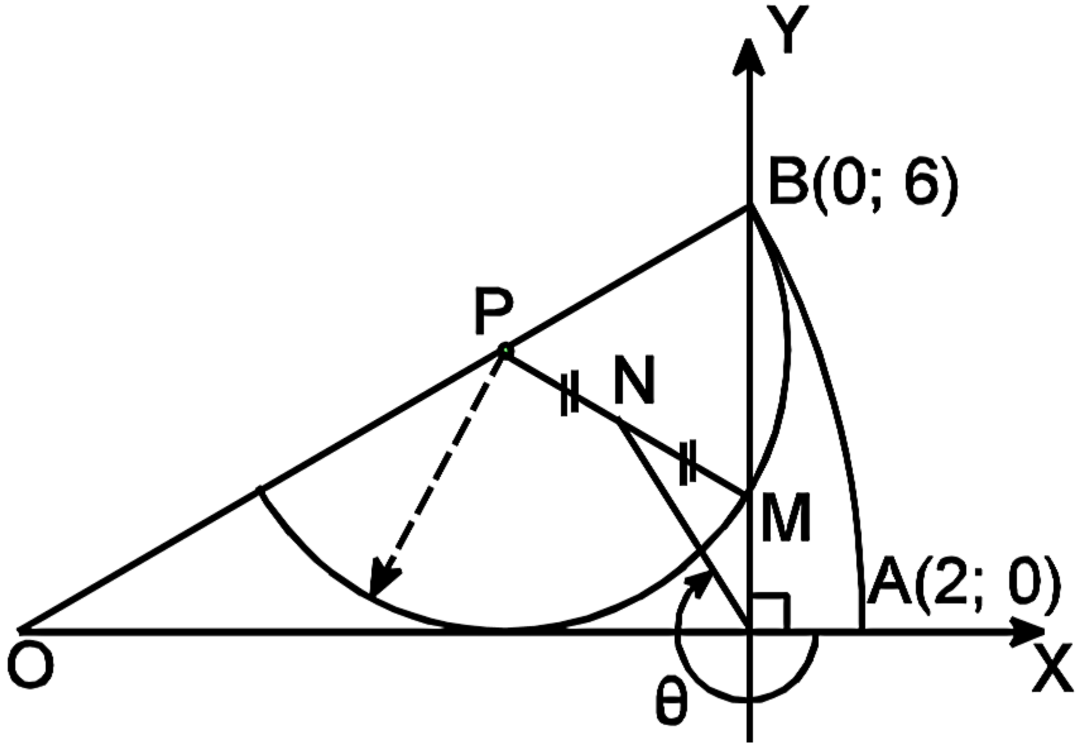
$$M = \frac{\sqrt{5}}{-1} + \frac{\sqrt{5}}{2}$$

$$\therefore M = -\frac{\sqrt{5}}{2}$$

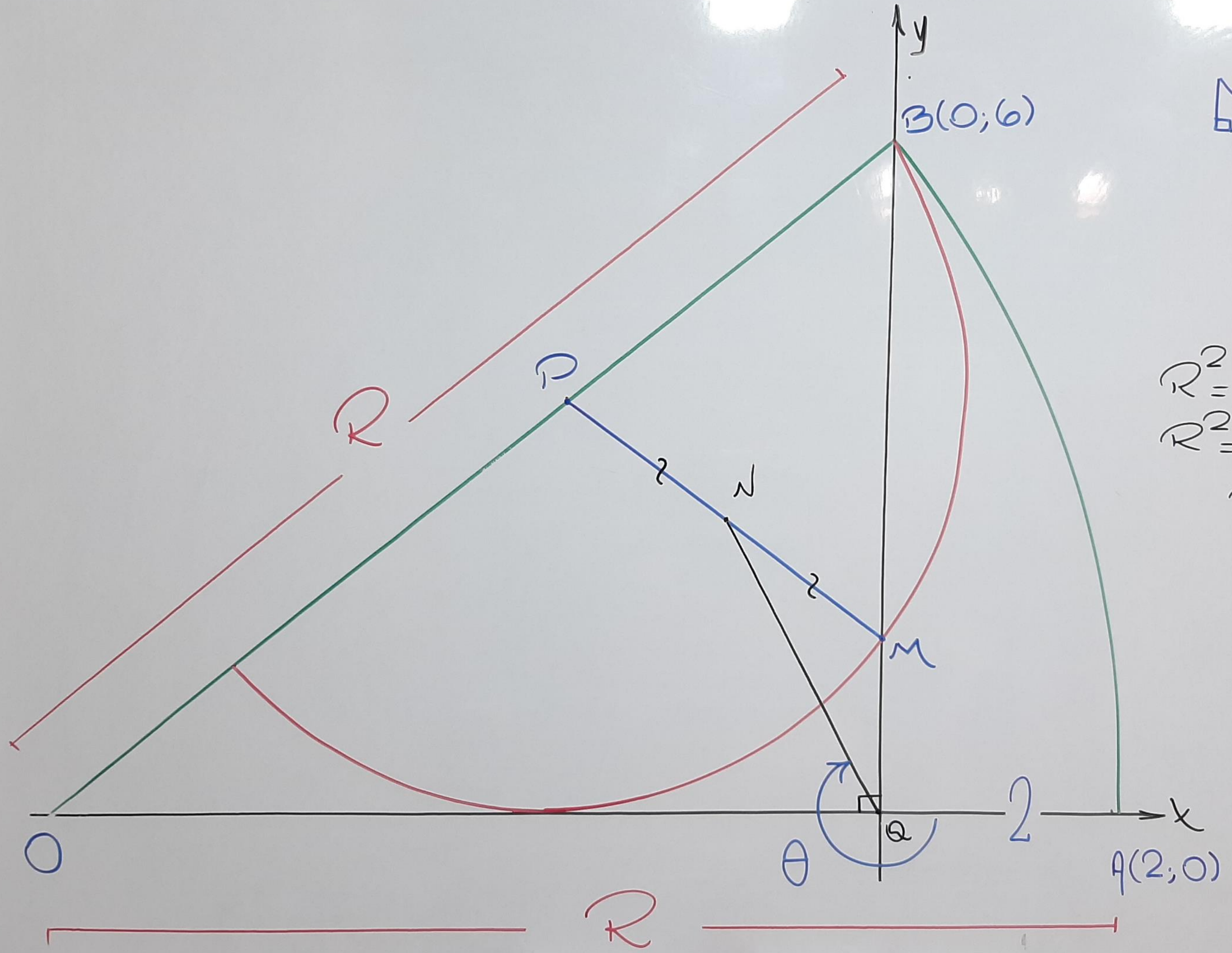
CLAVE D

Problema 2:

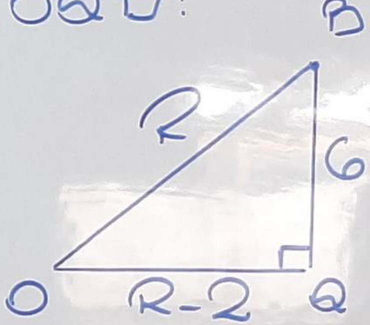
Del grafico, calcule 12Tanθ, siendo AOB un sector circular, además PN=NM



A) −30
B) −31
C) −32
D) −33
E) −21



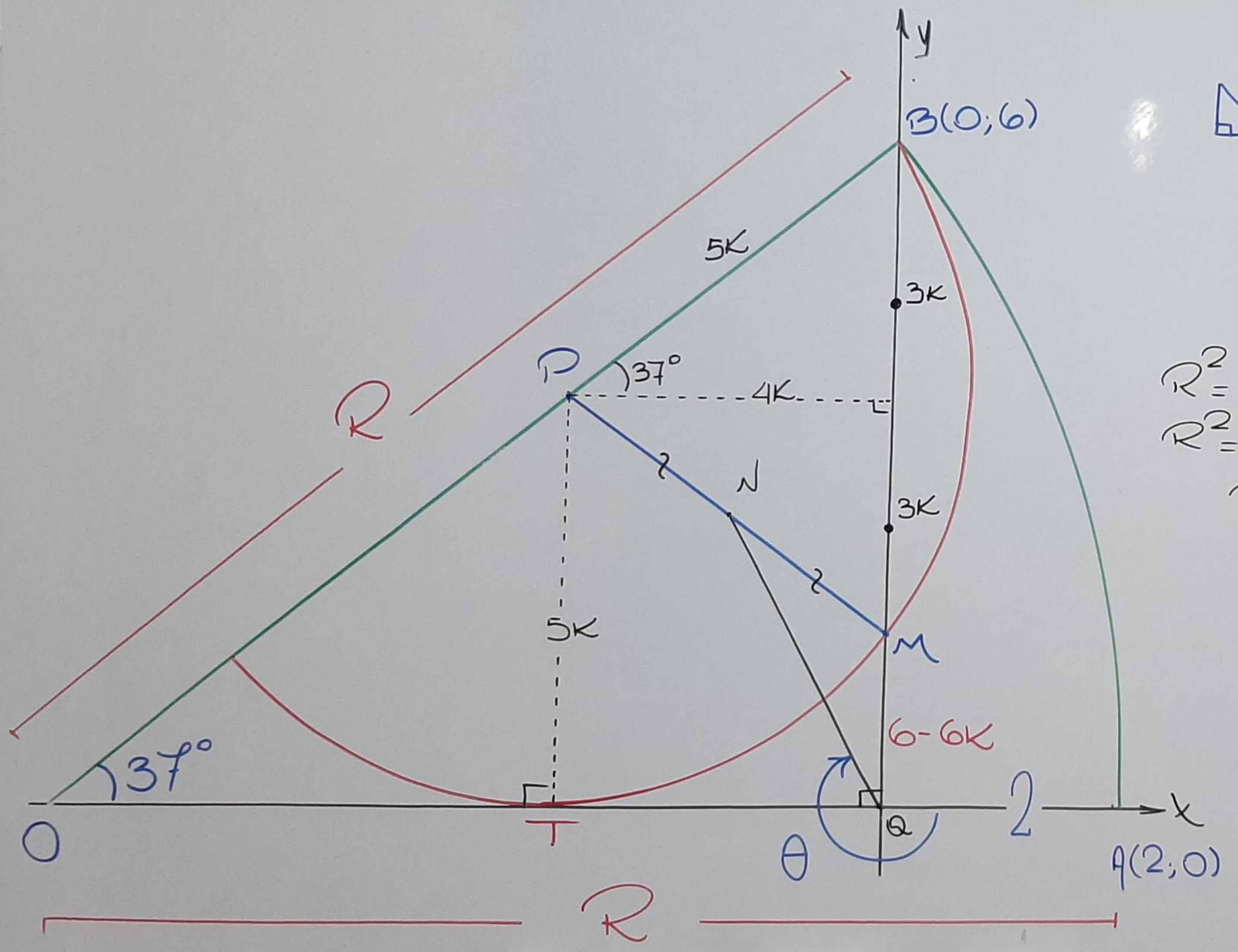
$\triangle OQB$:



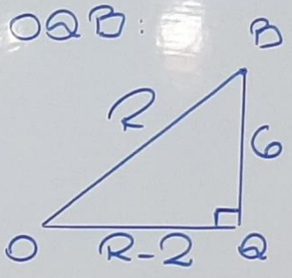
$$R^2 = (R-2)^2 + 6^2$$

$$R^2 = R^2 - 4R + 40$$

$$R = 10$$



$\triangle OQB$:



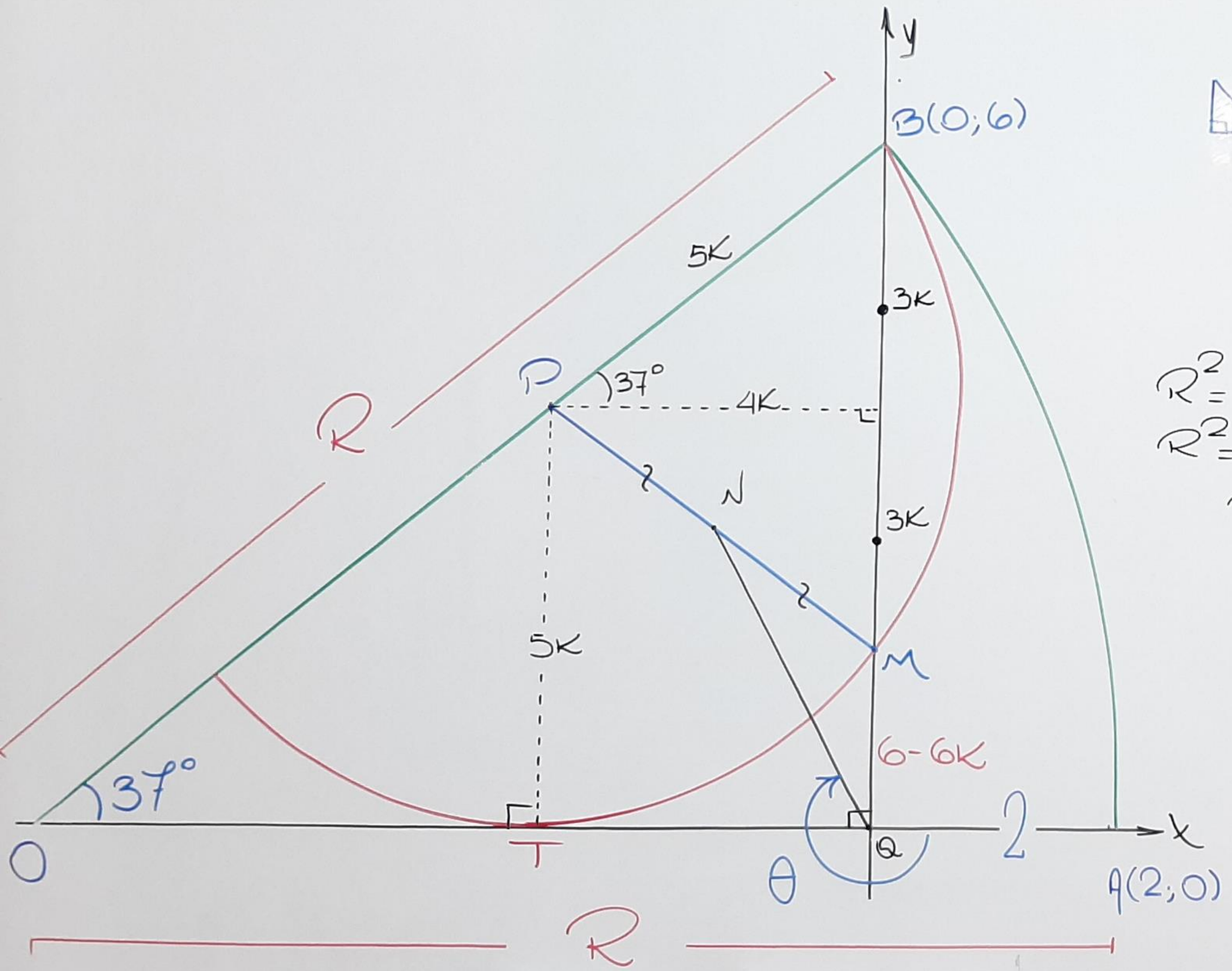
$$R^2 = (R-2)^2 + 6^2$$

$$R^2 = R^2 - 4R + 40$$

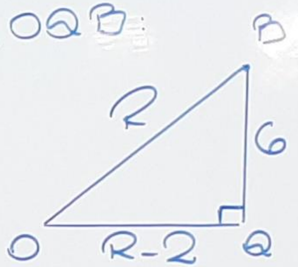
$$R = 10$$

$\rightarrow 5K + 3K = 6$

$K = \frac{3}{4} \longrightarrow \begin{cases} P\left(-3;\ \frac{15}{4}\right) \\ M\left(0,\ \frac{3}{2}\right) \end{cases}$



$\triangle OQB$:



$$R^2 = (R-2)^2 + 6^2$$
$$R^2 = R^2 - 4R + 40$$
$$R = 10$$

$$\to 3K + 3K = 6$$
$$K = \frac{3}{4} \longrightarrow \begin{cases} P\left(-3;\ \frac{15}{4}\right) \\ M\left(0;\ \frac{3}{2}\right) \end{cases}$$

$$\Rightarrow N\left(-\frac{3}{2};\ \frac{21}{8}\right)$$

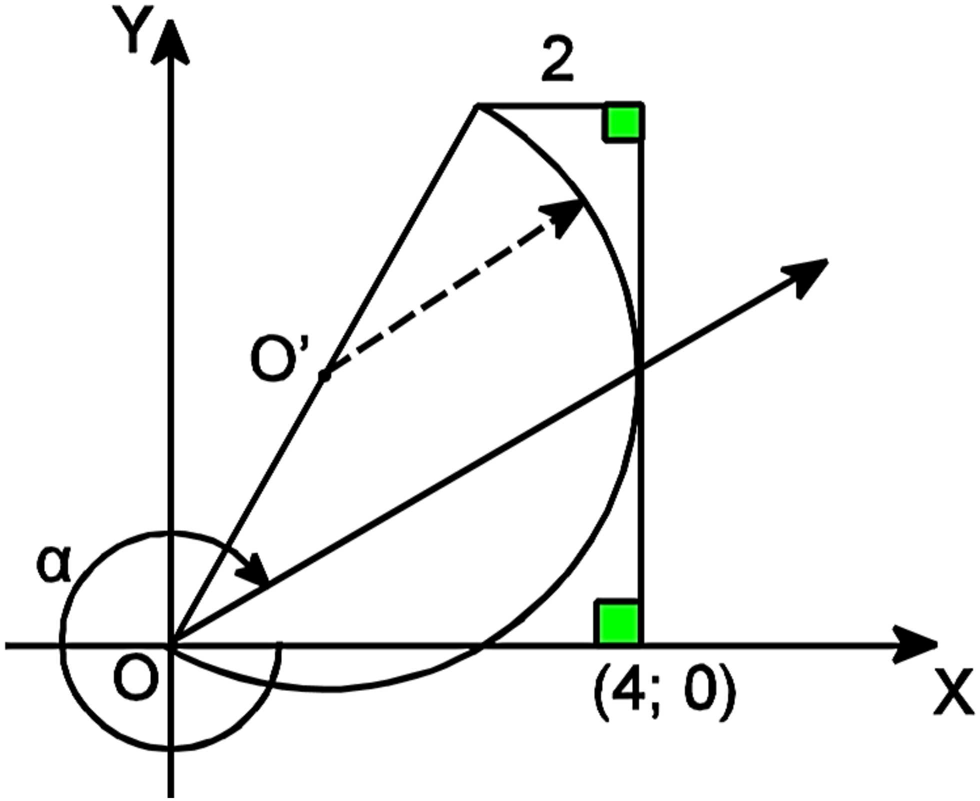
$$\tan\theta = \frac{y}{x} = \frac{\frac{21}{8}}{-\frac{3}{2}} = -\frac{7}{4}$$

$$\therefore 12\tan\theta = -21$$

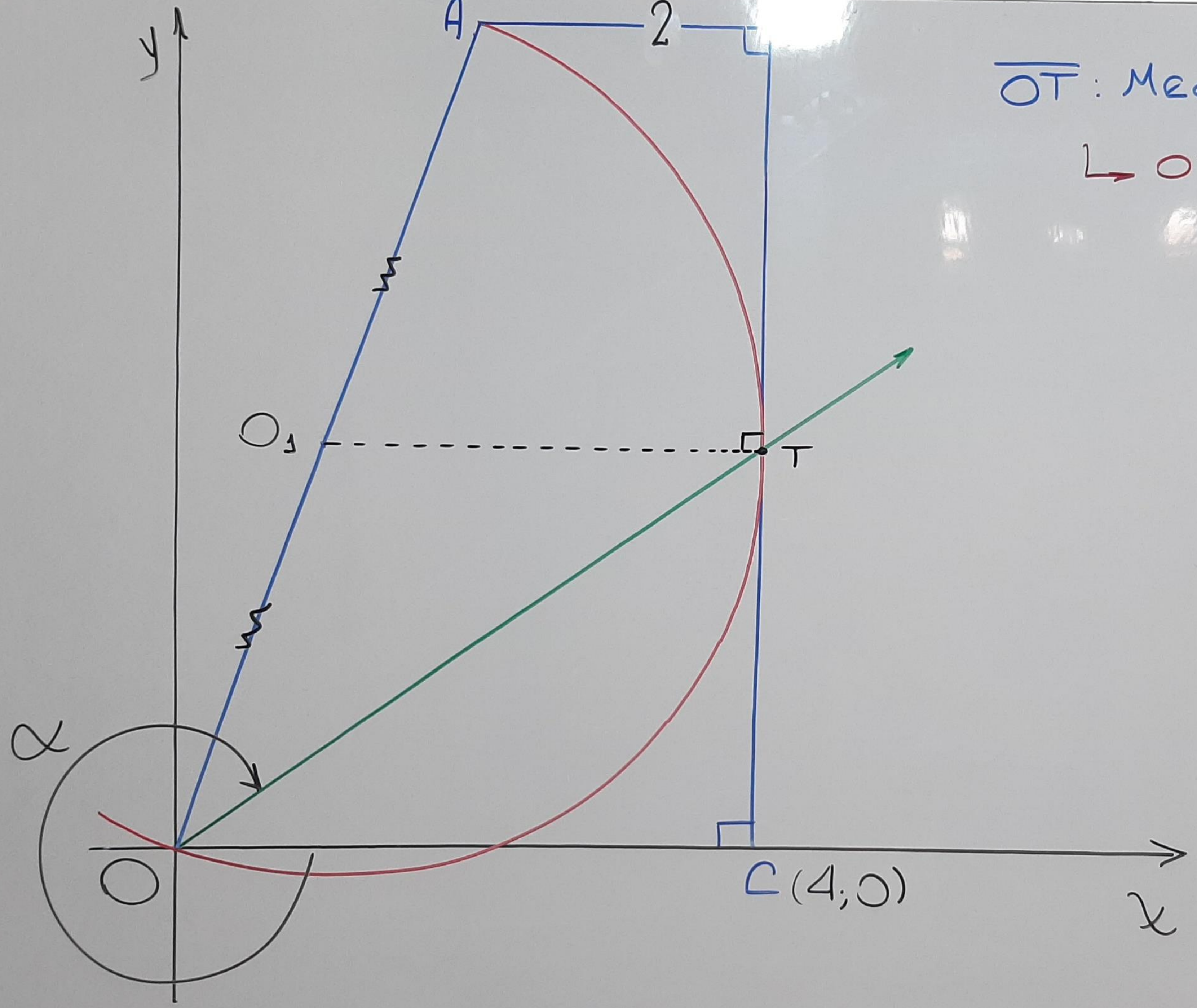
CLAVE E

Problema 3:

Del grafico, calcule Tanα

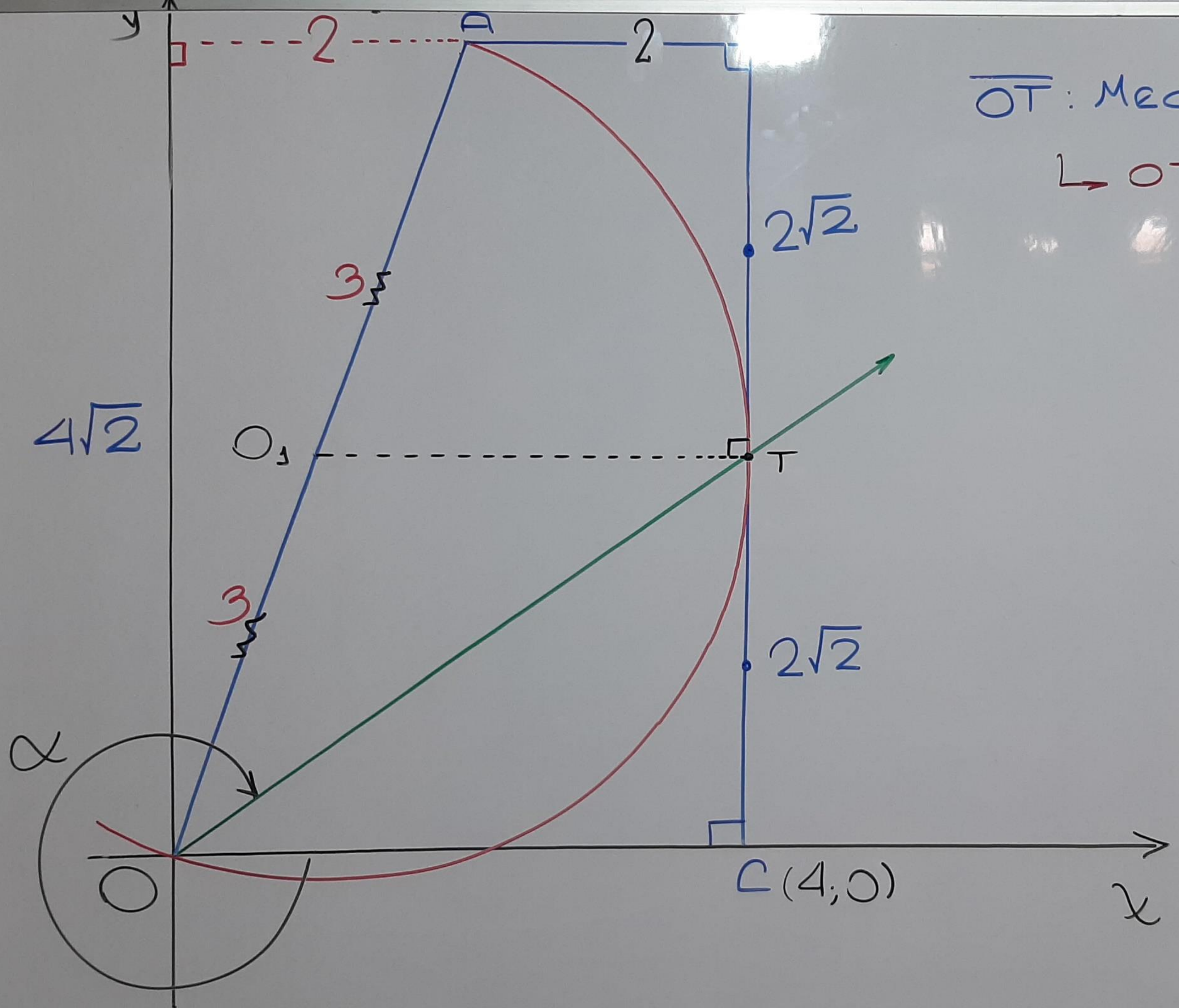


A) $\sqrt{2}$
B) $\sqrt{3}$
C) $\sqrt{5}$
D) $\sqrt{2}/2$
E) $\sqrt{2}/4$



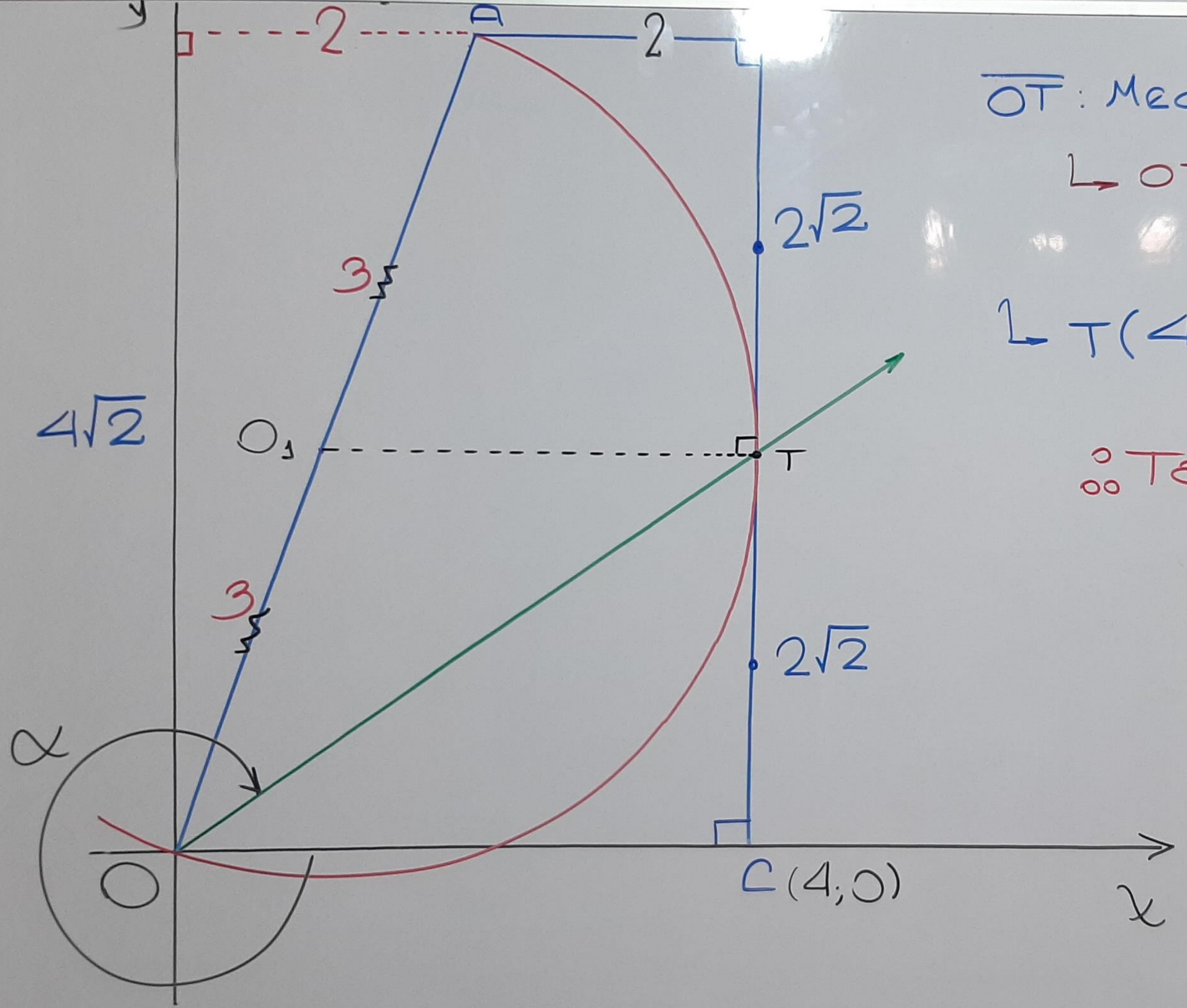
$\overline{OT}$: Mediana del $\square OABC$

$\rightarrow OT = 3$ (Radio)



$\overline{OT}$: Mediana del $\square$OABC

$\hookrightarrow$ OT = 3 (Radio)



$\overline{OT}$: Mediana del ⊿OABC

↳ $OT = 3$ (Radio)

↳ $T(4;\ 2\sqrt{2})$

$\therefore \operatorname{Tan}\alpha = \boxed{\dfrac{\sqrt{2}}{2}}$

CLAVE A

Problema 4:

Se tiene dos ángulos coterminales cuyo cociente es equivalente a 1/7 y que la suma de estos no es mayor que 500° ni menor de 400°. Calcule la suma de las tangentes de dichos ángulos.

A) 1 B) -1 C) 0

D) $-2\sqrt{3}$ E) $2\sqrt{3}$

Sean: α y β : ∡ Coterminales

i) $\dfrac{\alpha}{\beta} = \dfrac{1}{7}$

ii) $400^\circ < \alpha + \beta < 500^\circ$

Sean: α y β : $\measuredangle$ Coterminales

i) $\dfrac{\alpha}{\beta} = \dfrac{1}{7}$ $\begin{cases} \alpha = 1K \\ \beta = 7K \end{cases}$

ii) $400° < \alpha + \beta < 500°$

$$400° < 8K < 500°$$

$$50° < K < 62{,}5°$$

Sean: α y β : ∡ Coterminales → $\beta - \alpha = 360n°$

i) $\frac{\alpha}{\beta} = \frac{1}{7}$ → $\alpha = 1K$, $\beta = 7K$

$6K = 360n°$

$K = 60n°$

ii) $400° < \alpha + \beta < 500°$

$400° < 8K < 500°$

$50° < K < 62{,}5°$

$K = 60°$

Sean: α y β: ∡ Coterminales $\rightarrow$ $\beta - \alpha = 360n°$

i) $\dfrac{\alpha}{\beta} = \dfrac{1}{7}$ $\rightarrow$ $\alpha = 1K$, $\beta = 7K$

$6K = 360n°$

$K = 60n°$

ii) $400° < \alpha + \beta < 500°$

$400° < 8K < 500°$

$50° < K < 62{,}5°$

$K = 60°$

$\alpha = 60°$ $\quad$ $\beta = 420°$

CLAVE E

Piden: $\tan\alpha + \underbrace{\tan\beta}_{\tan\alpha}$

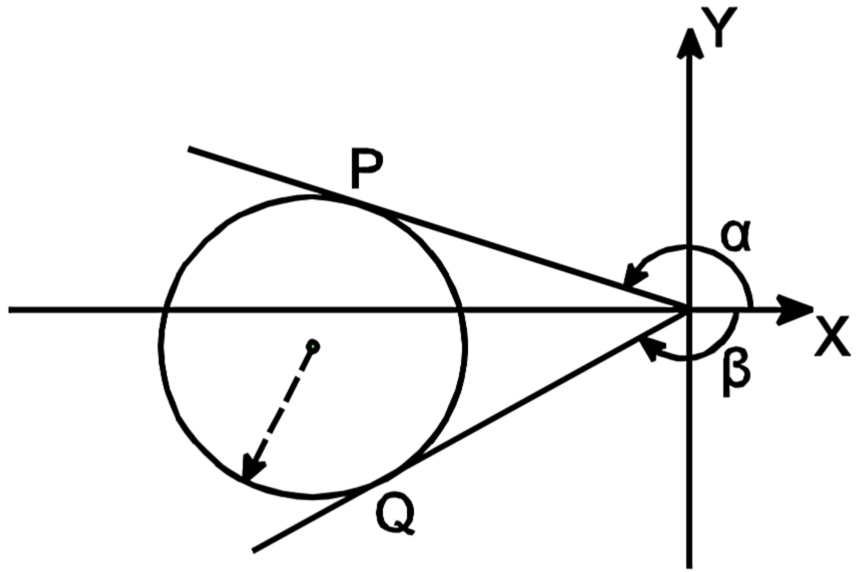
$2\tan\alpha$

$2\tan 60°$

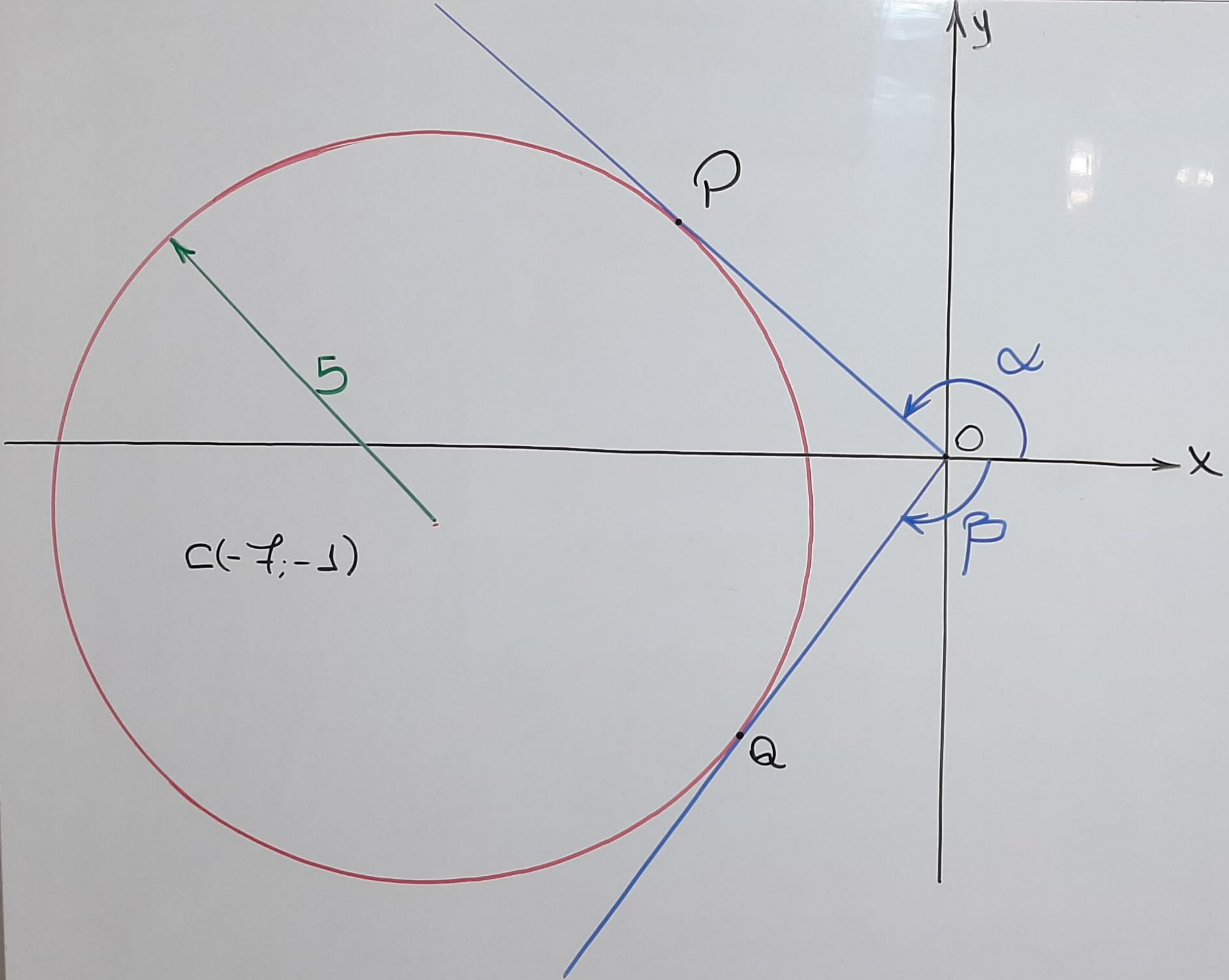
$2\sqrt{3}$

Problema 5:

En el grafico mostrado el centro de la circunferencia de radio 5 es $(-7;-1)$. Se pide: $M = \text{Sen}\alpha\text{Sec}\beta$
P y Q son puntos de tangencia.

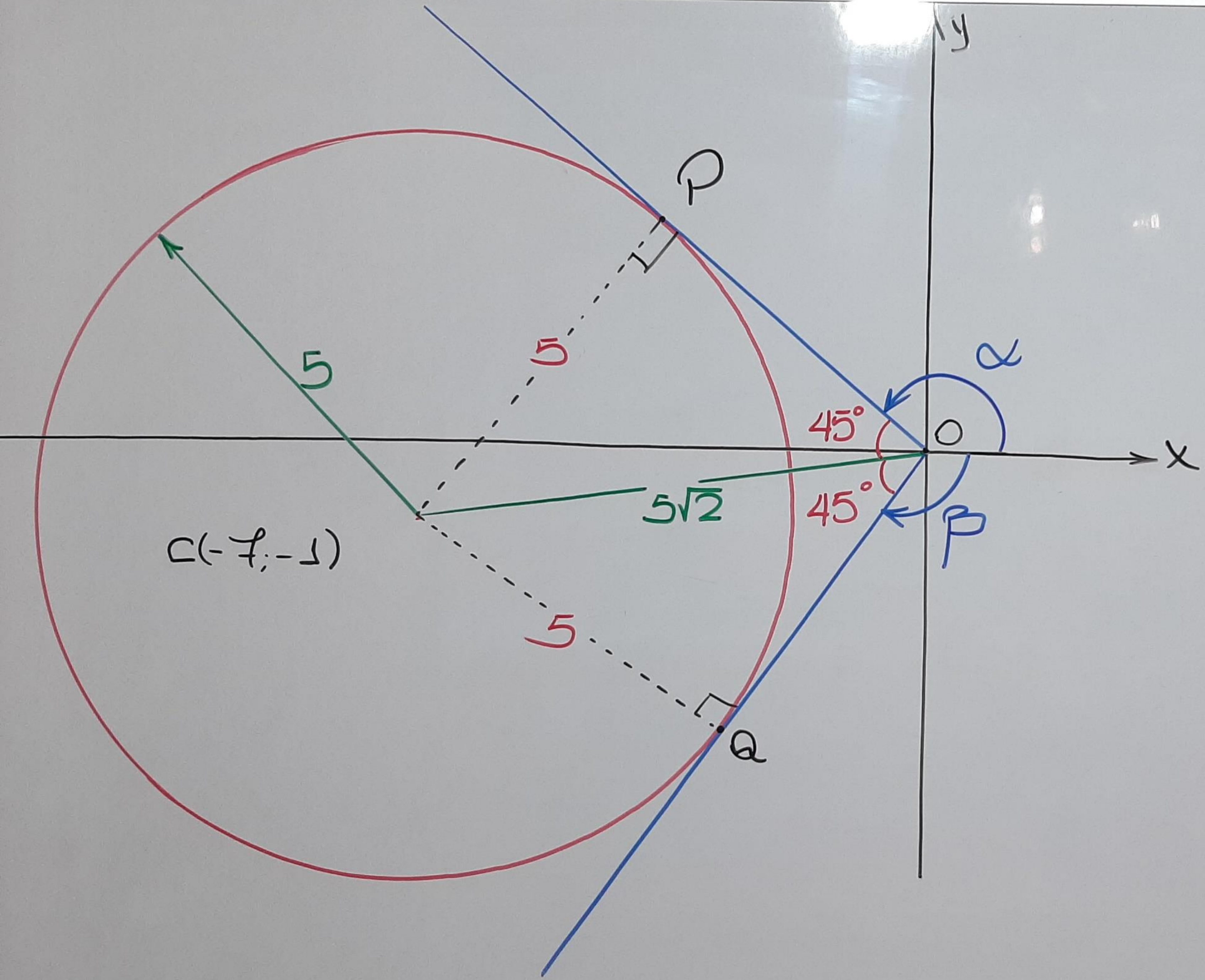


A) -1 B) -2 C) $-1/2$
D) $-1/3$ E) $-2/3$



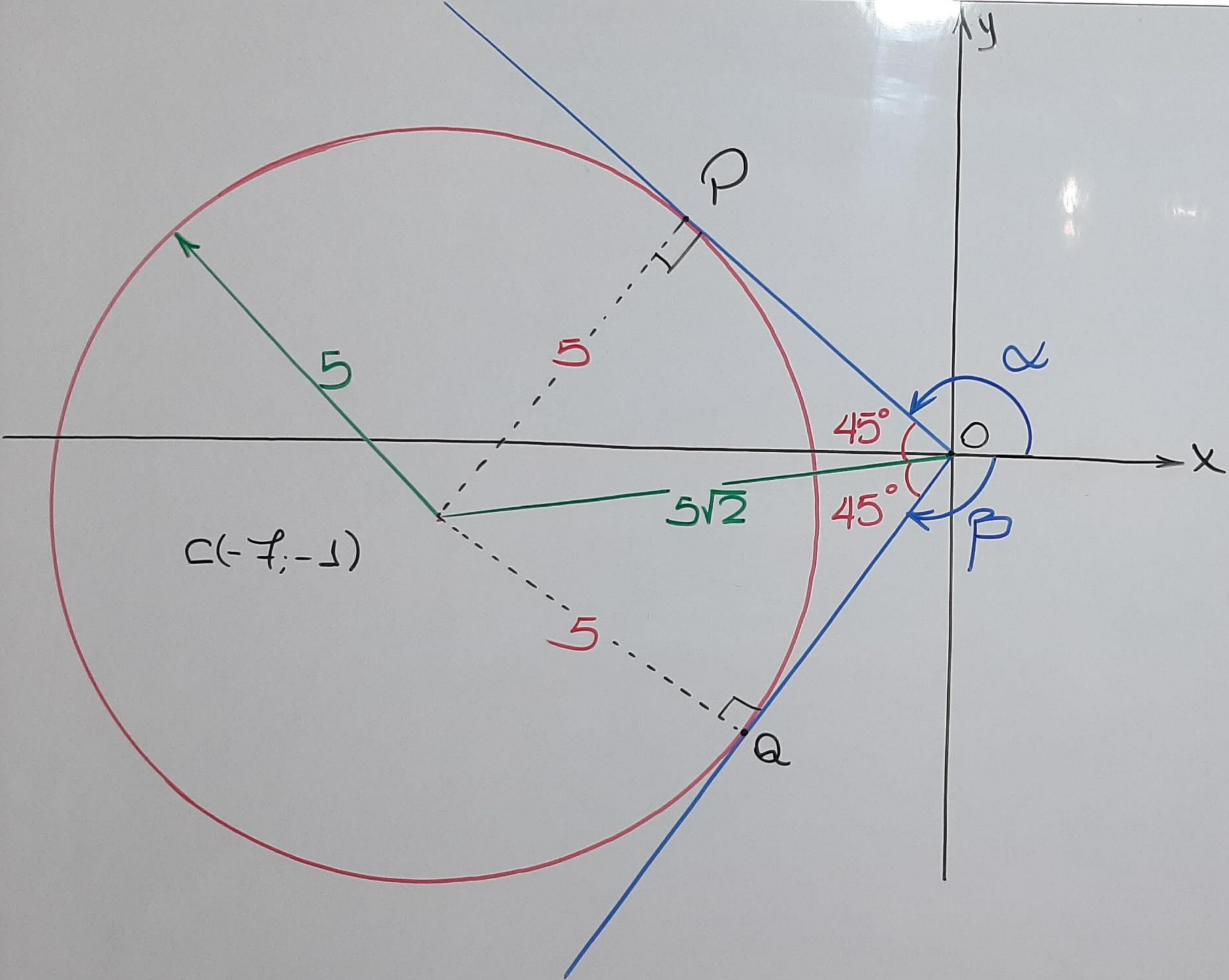
i) $OC = \sqrt{(-7)^2+(-1)^2}$

$OC = 5\sqrt{2}$



i) $OC = \sqrt{(-7)^2 + (-1)^2}$

$OC = 5\sqrt{2}$

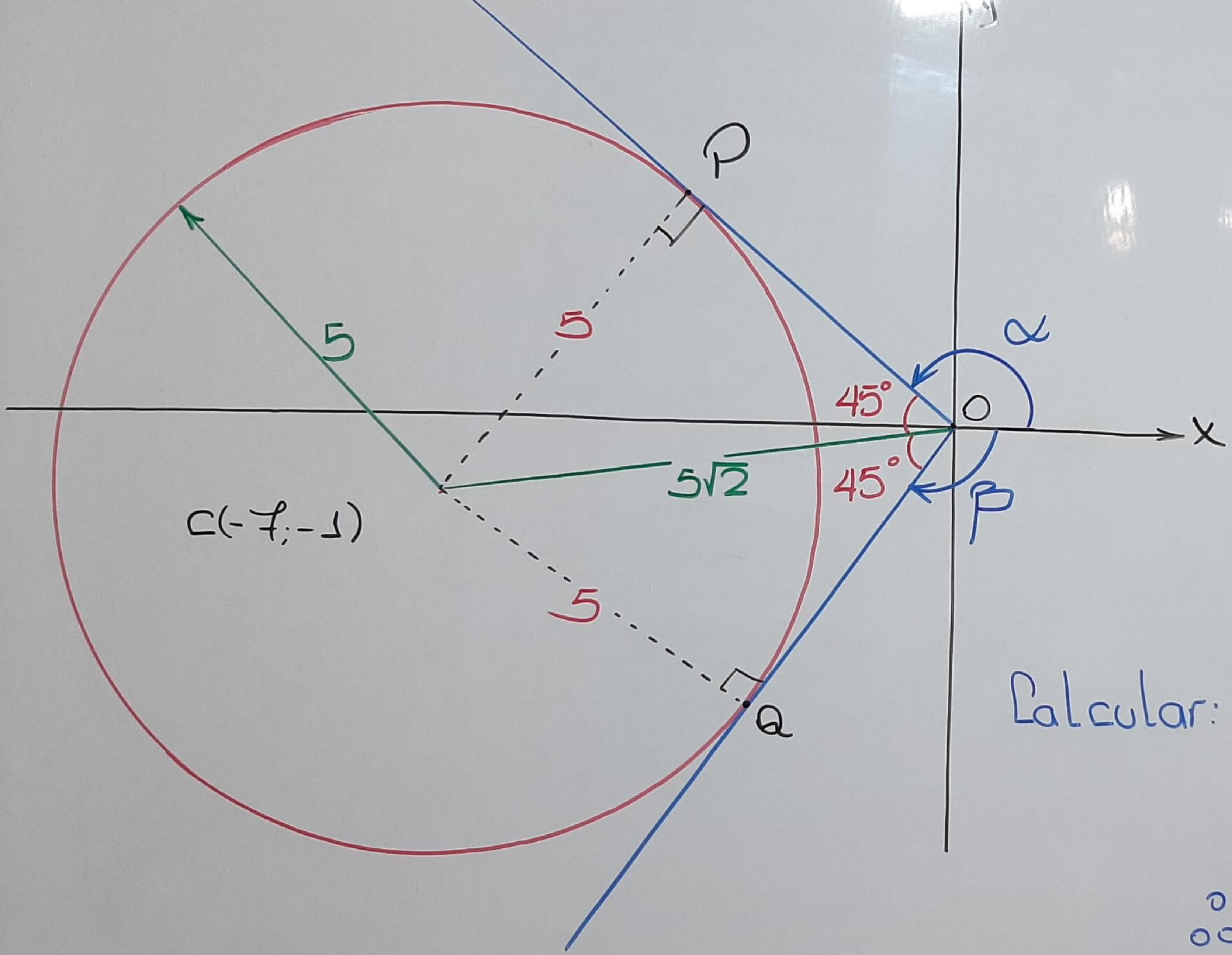


i) $OC = \sqrt{(-7)^2 + (-1)^2}$

$OC = 5\sqrt{2}$

ii) $\alpha + 90^\circ - \beta = 360^\circ$

$\alpha = 270^\circ + \beta$



i) $OC = \sqrt{(-7)^2 + (-1)^2}$

$OC = 5\sqrt{2}$

ii) $\alpha + 90° - \beta = 360°$

$\alpha = 270° + \beta$

$\text{Sen}\,\alpha = \text{Sen}(270° + \beta)$

$\text{Sen}\,\alpha = -\text{Cos}\,\beta$

Calcular:

$M = \underbrace{\text{Sen}\,\alpha}\,.\,\text{Sec}\,\beta$

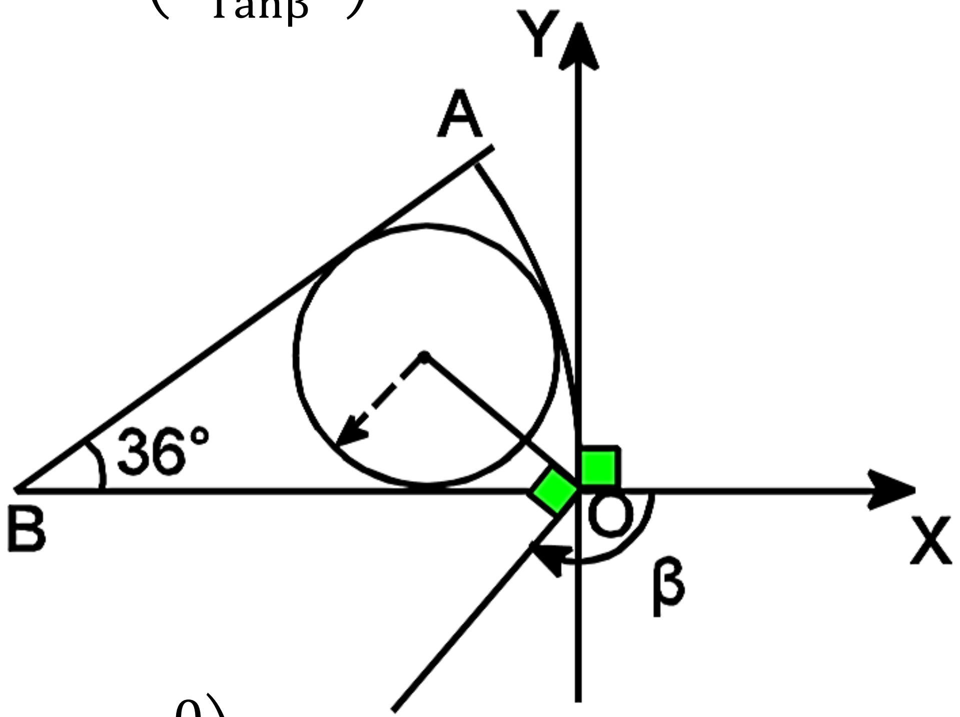
$M = -\text{Cos}\,\beta\,.\,\text{Sec}\,\beta$

$\therefore M = -1$

CLAVE A

Problema 6:

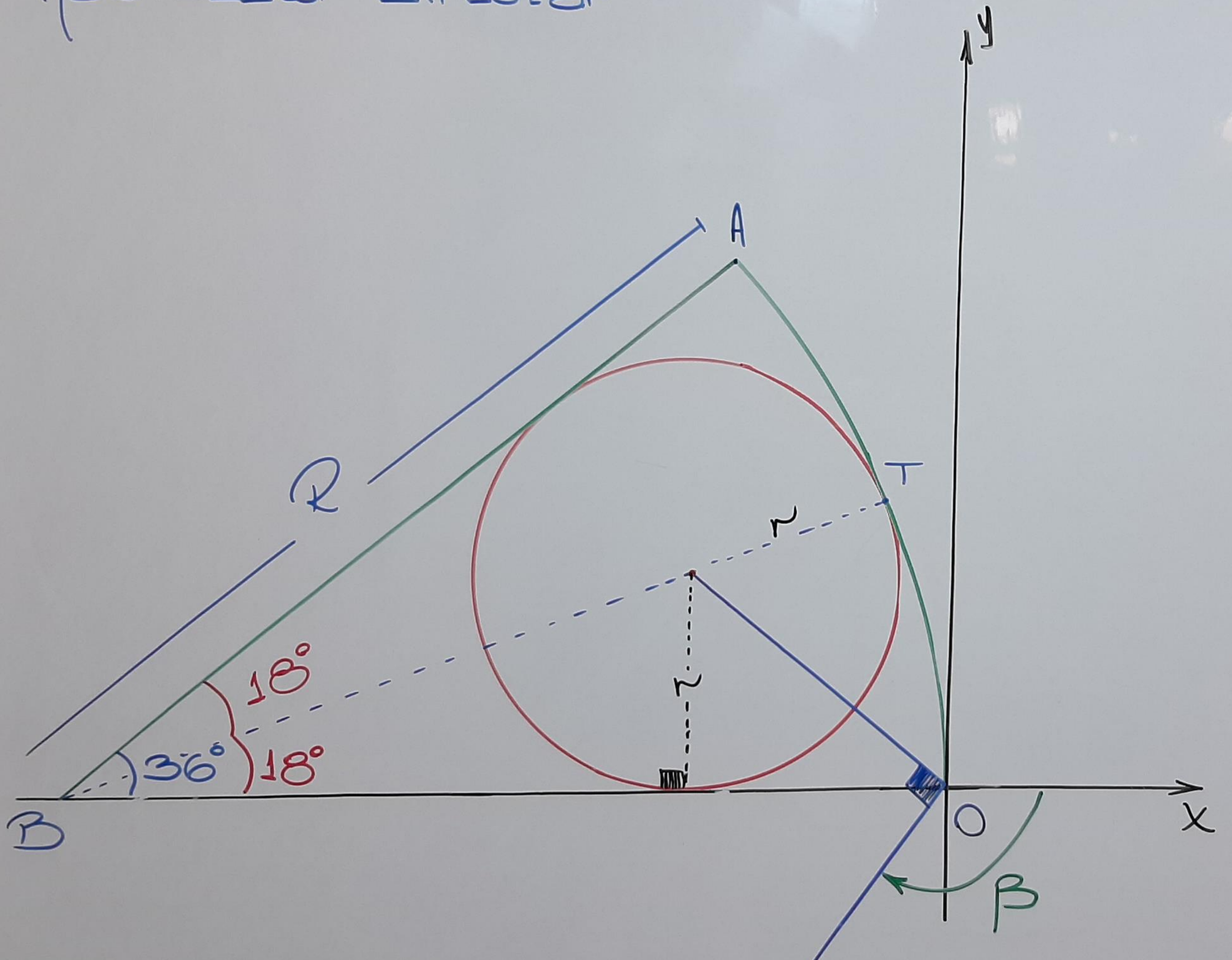
Del grafico, calcular: $R = \left(\frac{1+\text{Tan}9°}{\text{Tan}\beta}\right)$,AOB: Sector circular con centro en B.



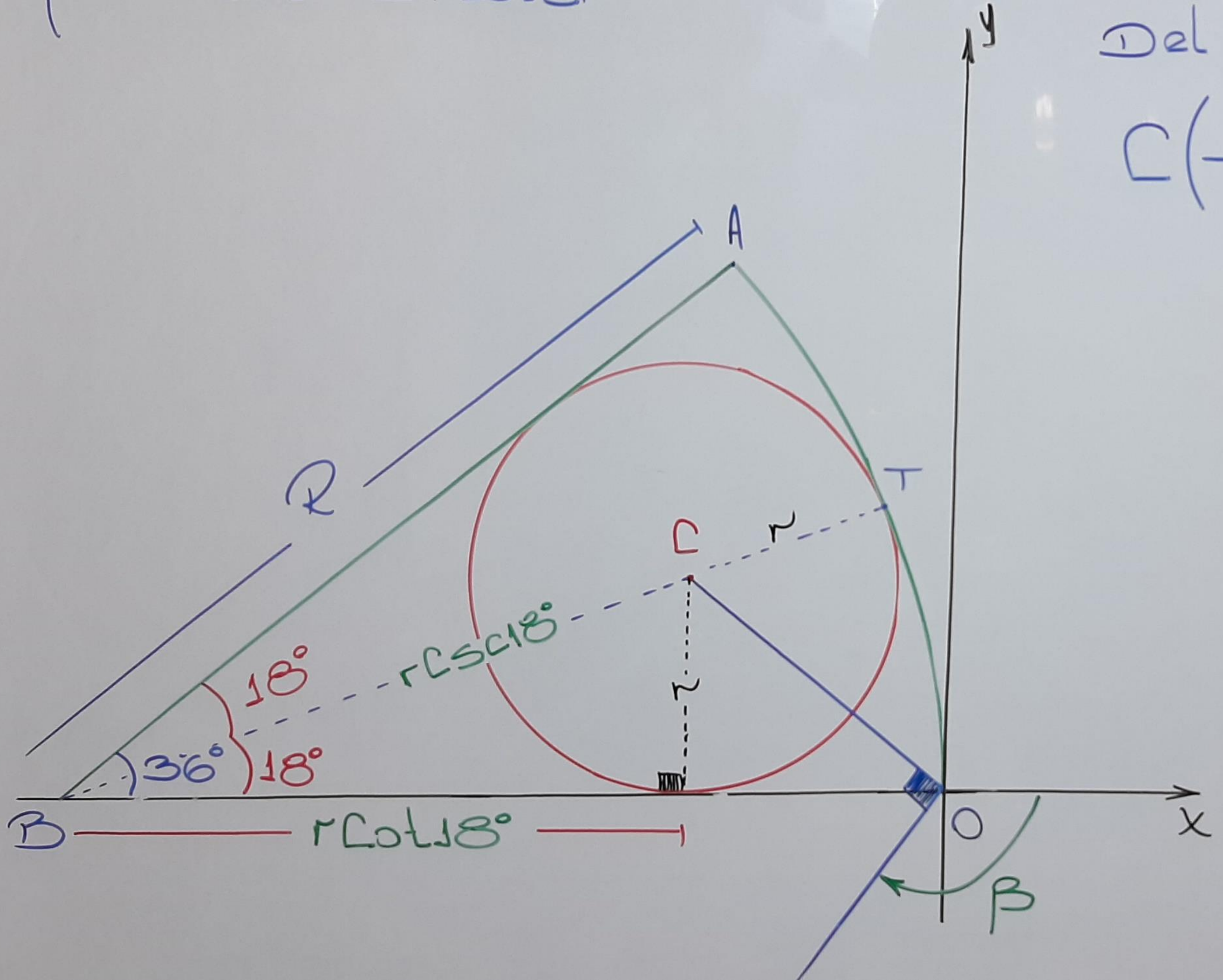
$\left(\text{Nota: Csc}\theta - \text{Cot}\theta = \text{Tan}\frac{\theta}{2}\right)$

A) Tan18°
B) Tan9°
C) 1
D) 2
E) −1

ABO: Sector Circular



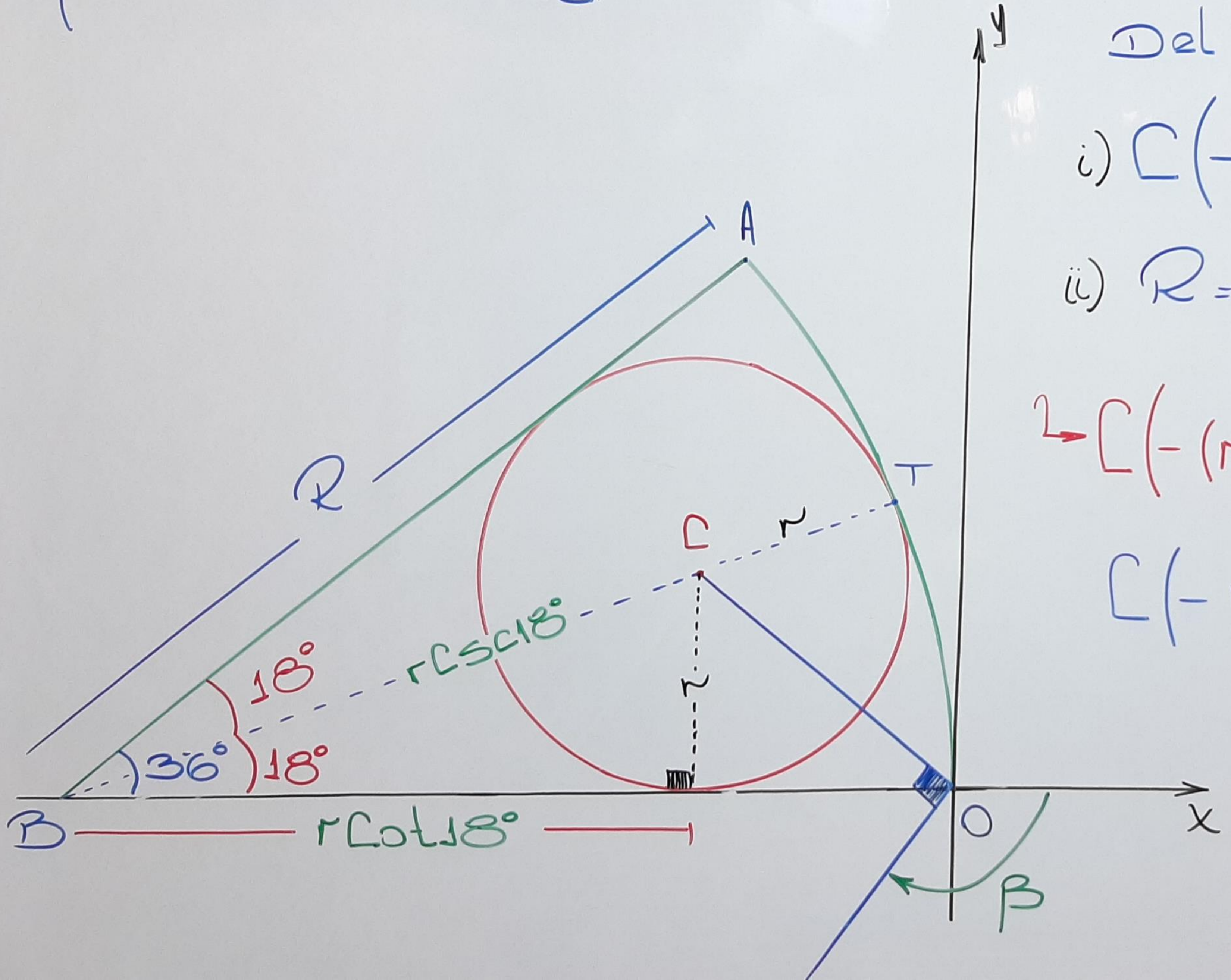
AB0: Sector Circular



Del gráfico:

$$C\left(-(R - r\cot 18°);\ r\right)$$

ABO: Sector Circular



Del gráfico:

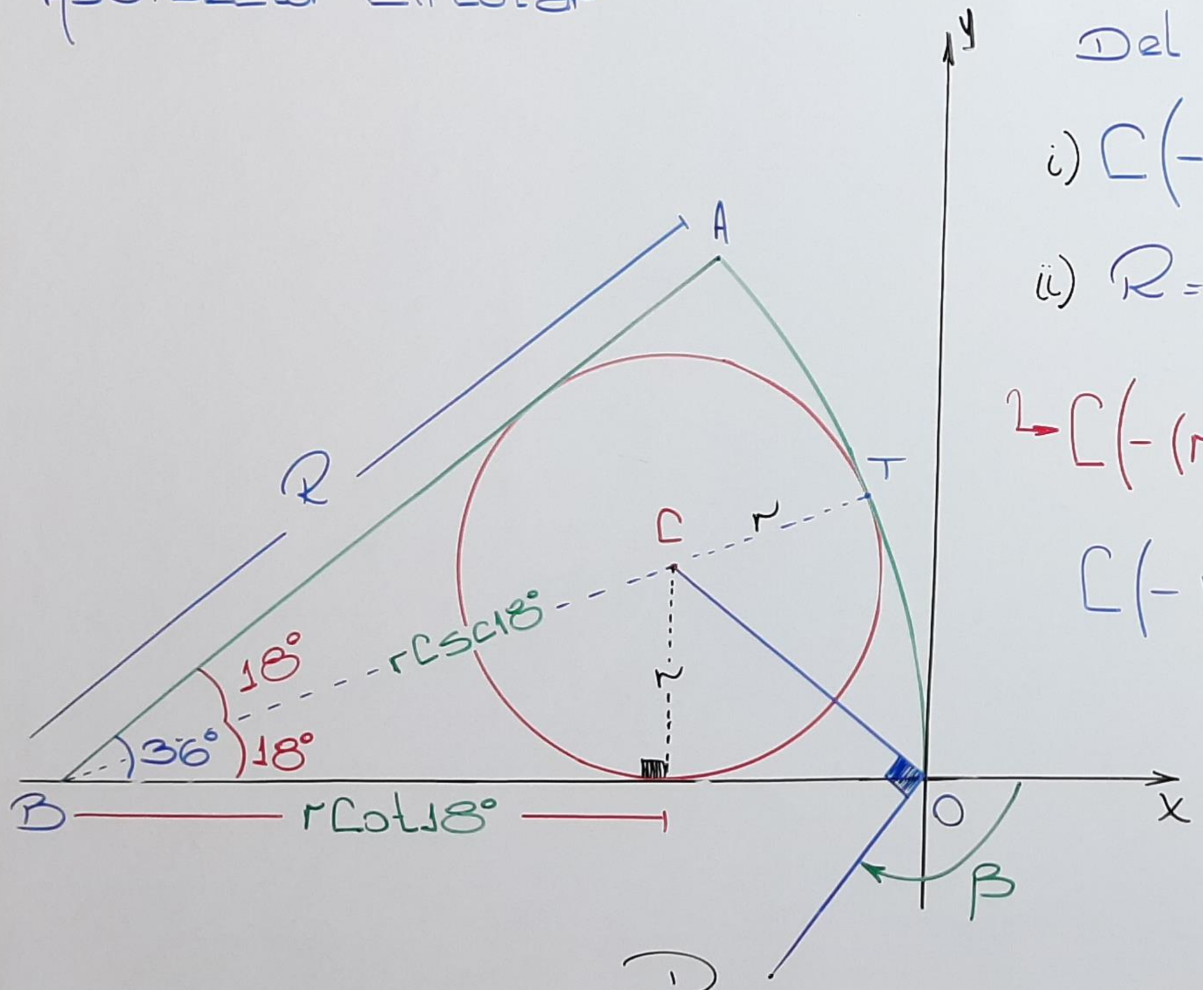
i) $C\left(-(R - r\cot 18°); r\right)$

ii) $R = r\csc 18° + r$

$\rightarrow C\left(-(\underbrace{r\csc 18° + r - r\cot 18°}); r\right)$

$C\left(-r(\tan 9° + 1); r\right)$

AOB: Sector Circular



Del gráfico:

i) $C(-(R-r\cot 18°); r)$

ii) $R = r\csc 18° + r$

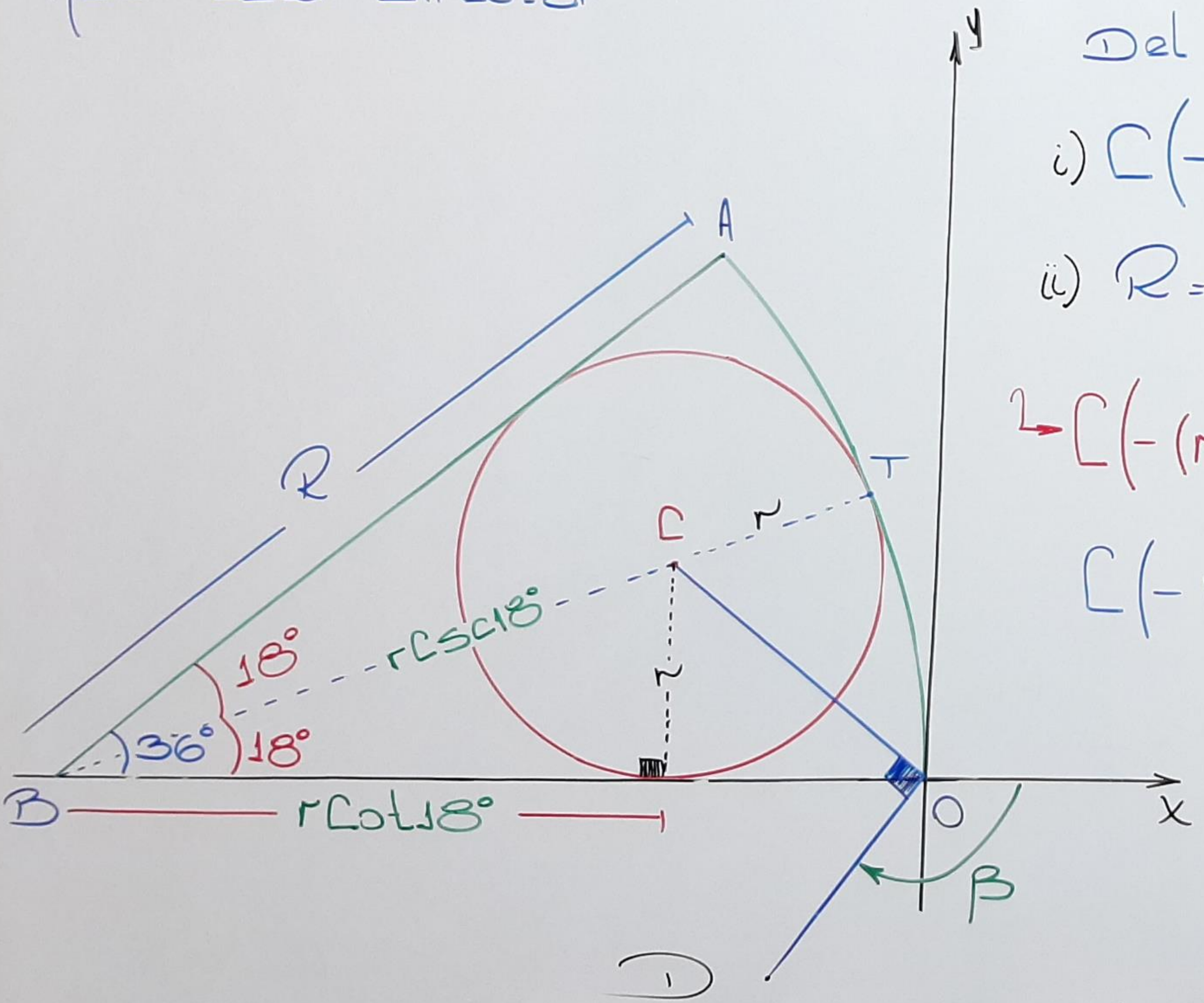
$\to C(-(\underbrace{r\csc 18° + r - r\cot 18°}); r)$

$C(-r(\tan 9° + 1); r)$

Por coord. ortogonales:

$D(-r; -r(\tan 9° + 1))$

AOB: Sector Circular



Del gráfico:

i) $C(-(R - r\cot 18°);\ r)$

ii) $R = r\csc 18° + r$

$\to C(-(\underbrace{r\csc 18° + r - r\cot 18°});\ r)$

$C(-r(\tan 9° + 1);\ r)$

Por coord. ortogonales:

$D(-r\ ;\ -r(\tan 9° + 1))$

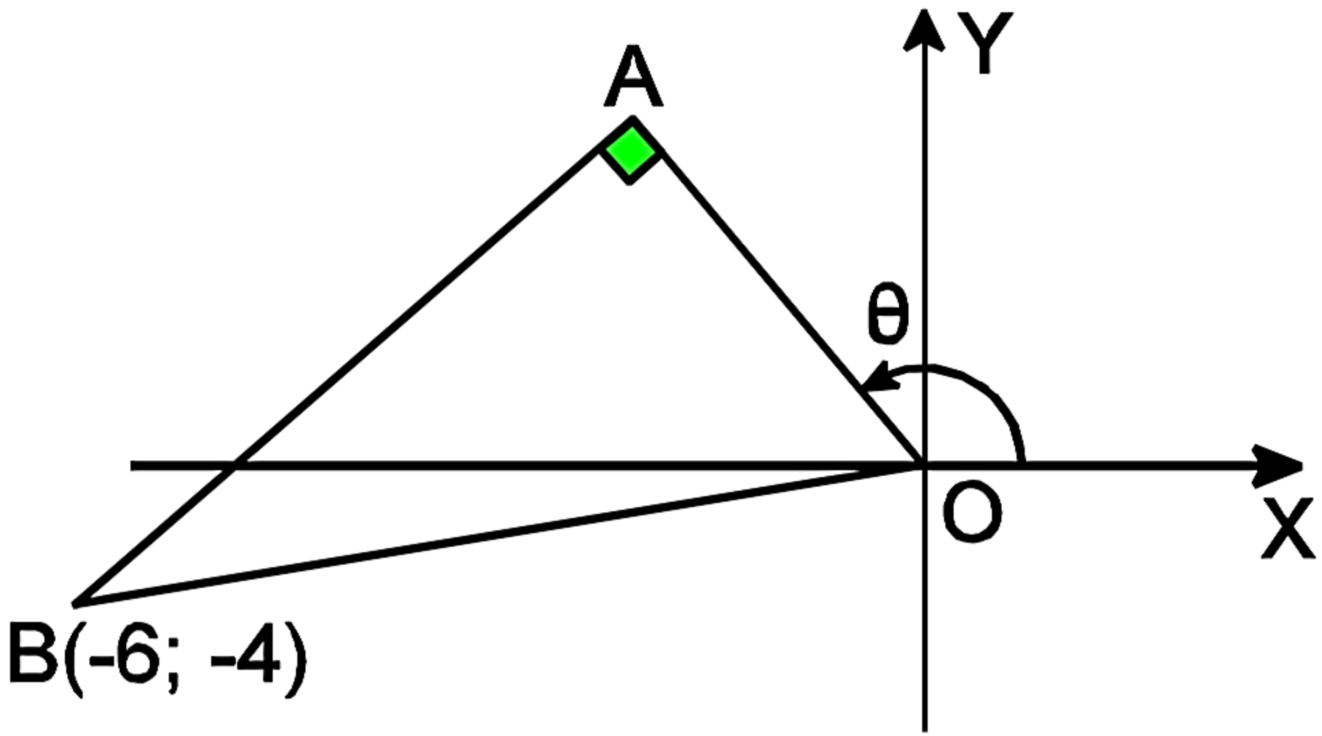
$\to \tan\beta = \dfrac{\cancel{-r}(\tan 9° + 1)}{\cancel{-r}}$

$\therefore \dfrac{1 + \tan 9°}{\tan\beta} = 1$

CLAVE C

Problema 7:

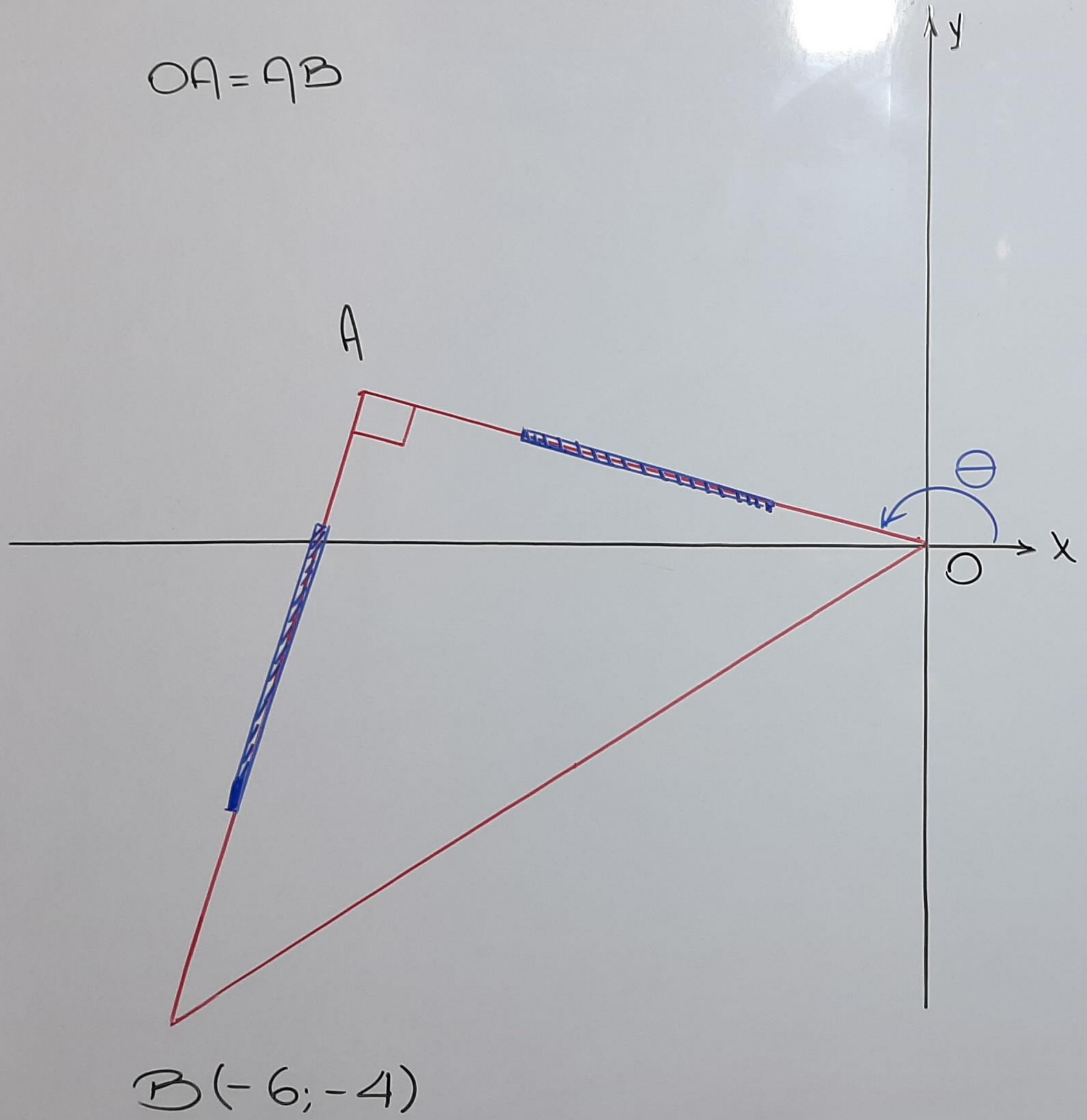
Calcular: $\text{Sen}\theta . \text{Cos}\theta$, si: $OA=AB$



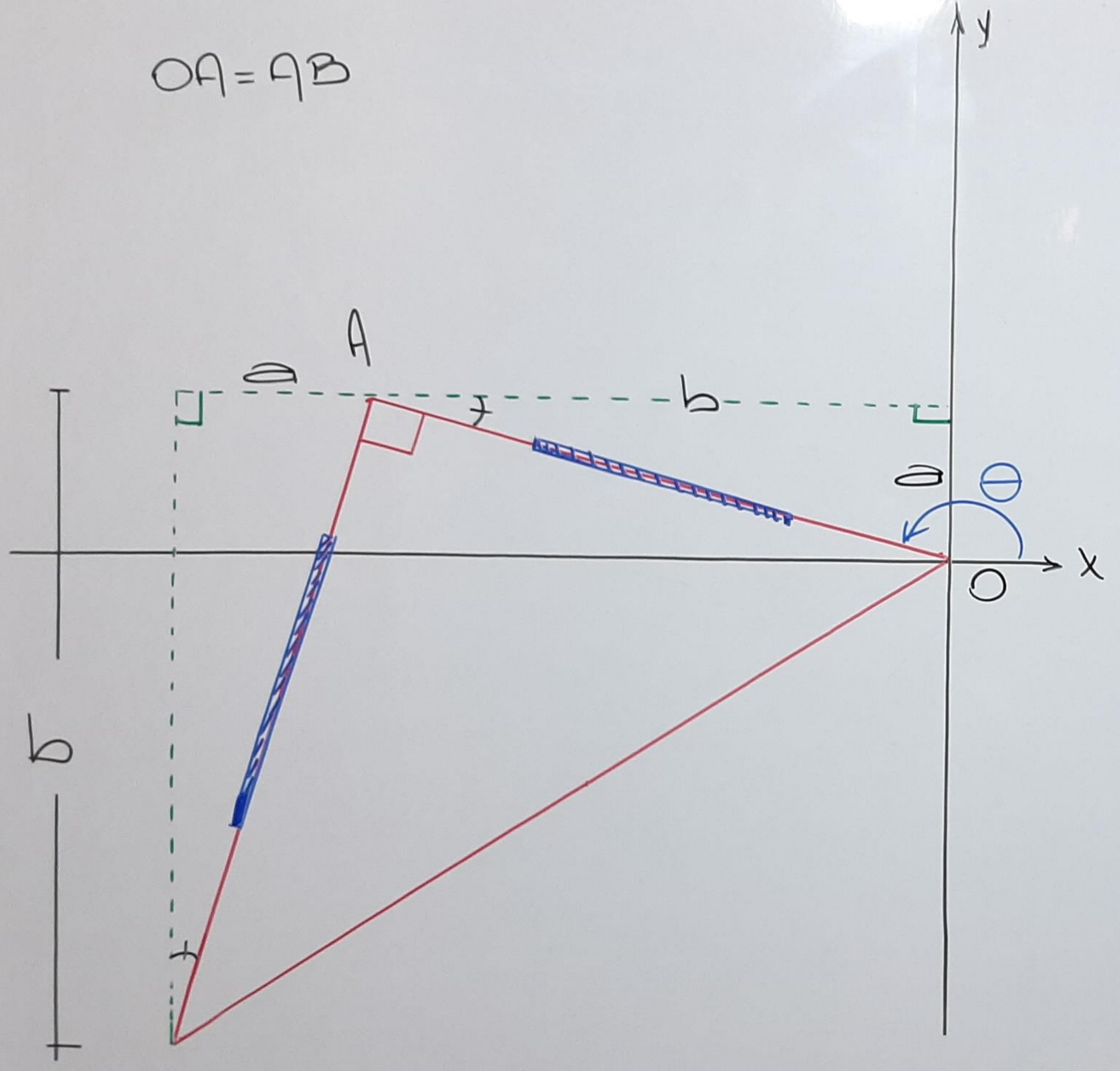
A) $-0,1$ B) $-0,2$ C) $-0,3$

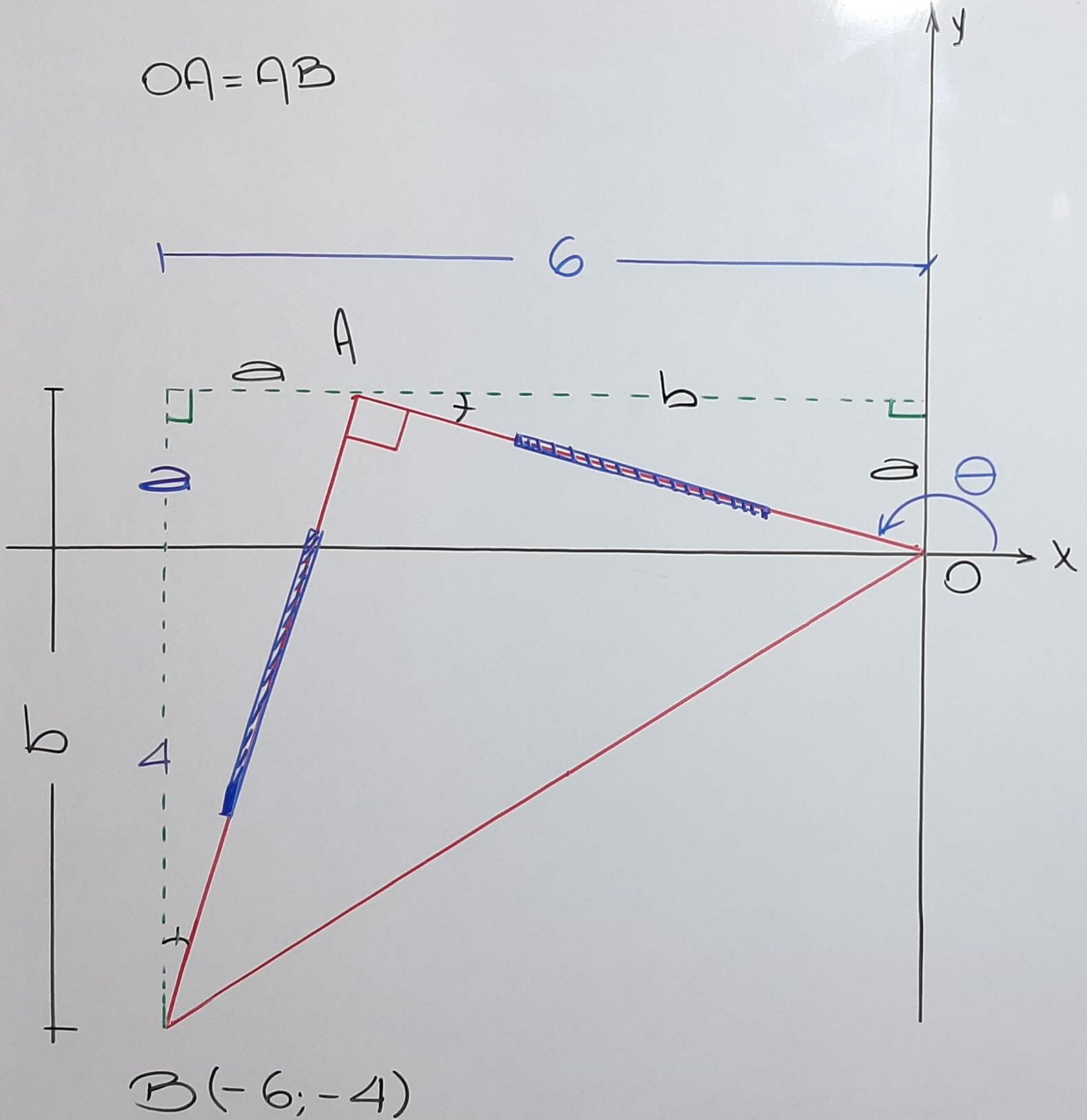
D) $-0,4$ E) $-0,5$

OA = AB



$OA = AB$

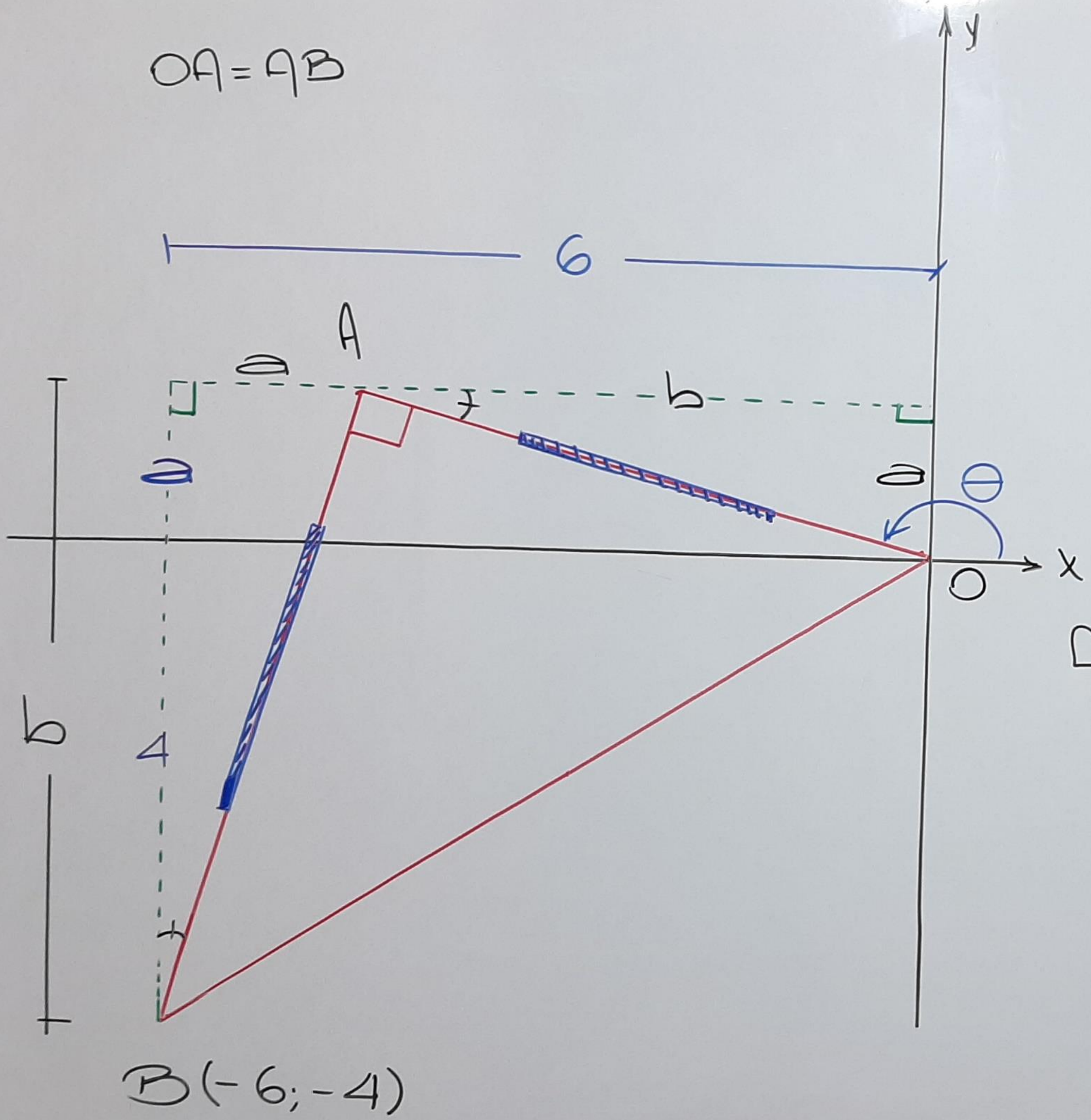




Del gráfico:

$$\begin{array}{r} a+b=6 \\ b-a=4 \\ \hline b=5 \rightarrow a=1 \end{array} \quad +$$

$OA = AB$



Del gráfico:

$$\begin{aligned} a + b &= 6 \\ b - a &= 4 \\ \hline b = 5 &\wedge a = 1 \end{aligned}$$

$A(-5;\ 1) \wedge r_A = \sqrt{26}$

Calcular: $\operatorname{sen}\theta \times \cos\theta$

$$\frac{1}{\sqrt{26}} \times \frac{-5}{\sqrt{26}}$$

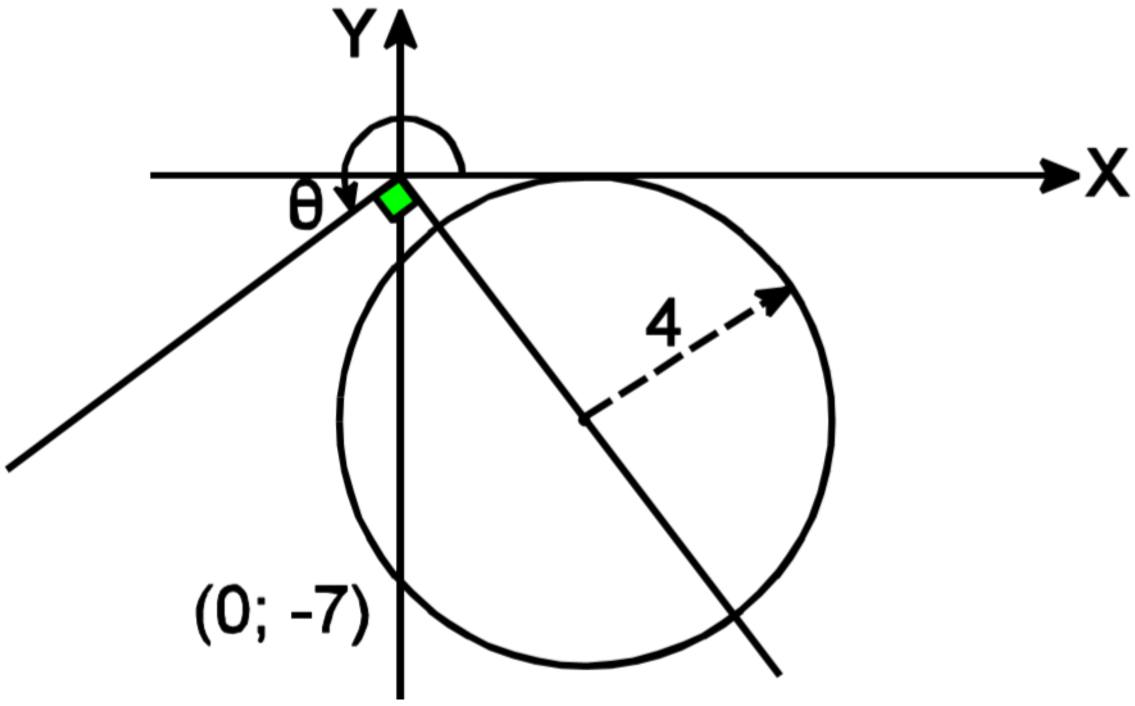
$$-5/26$$

$$\underline{-0,19}$$

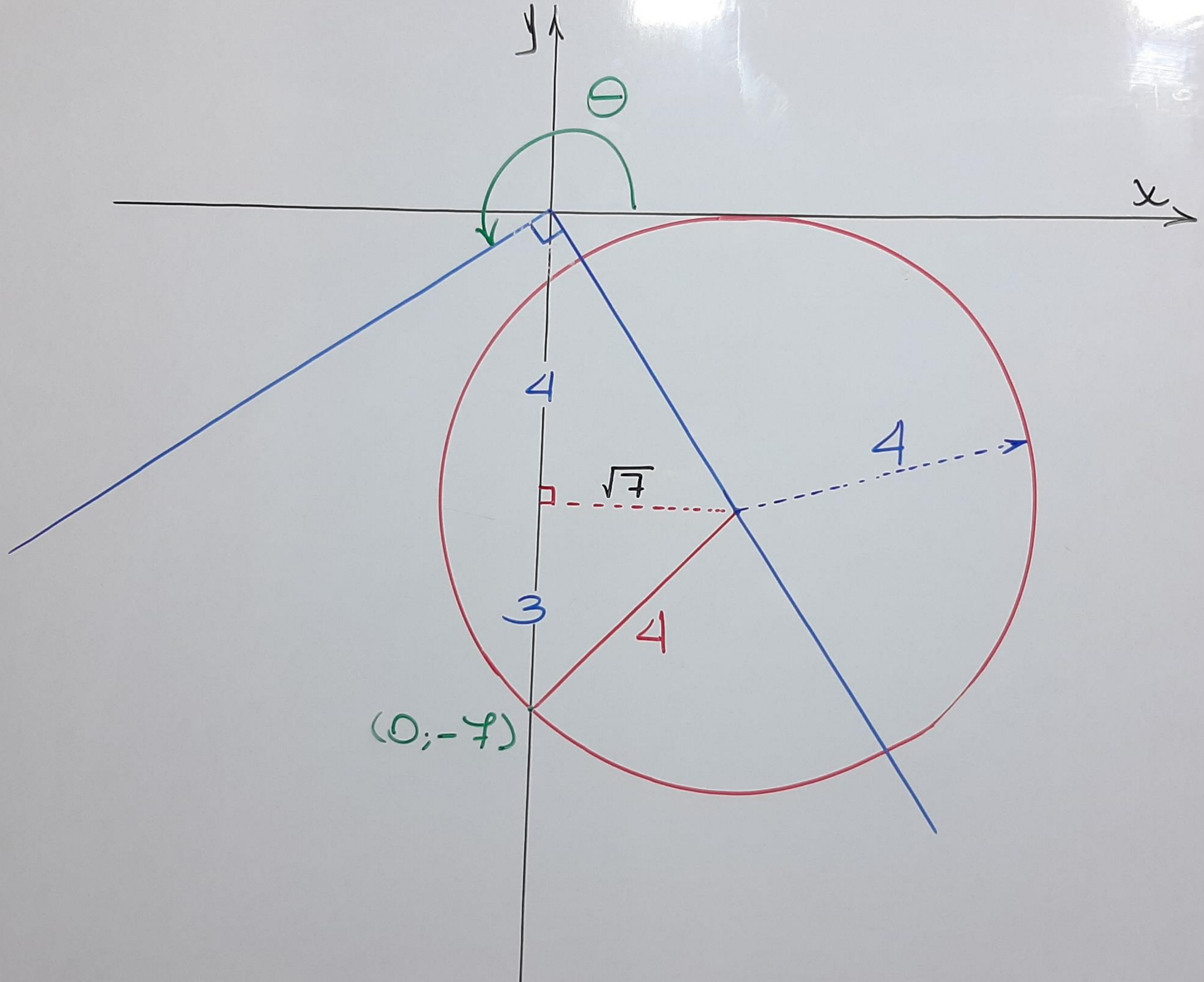
CLAVE B

Problema 8:

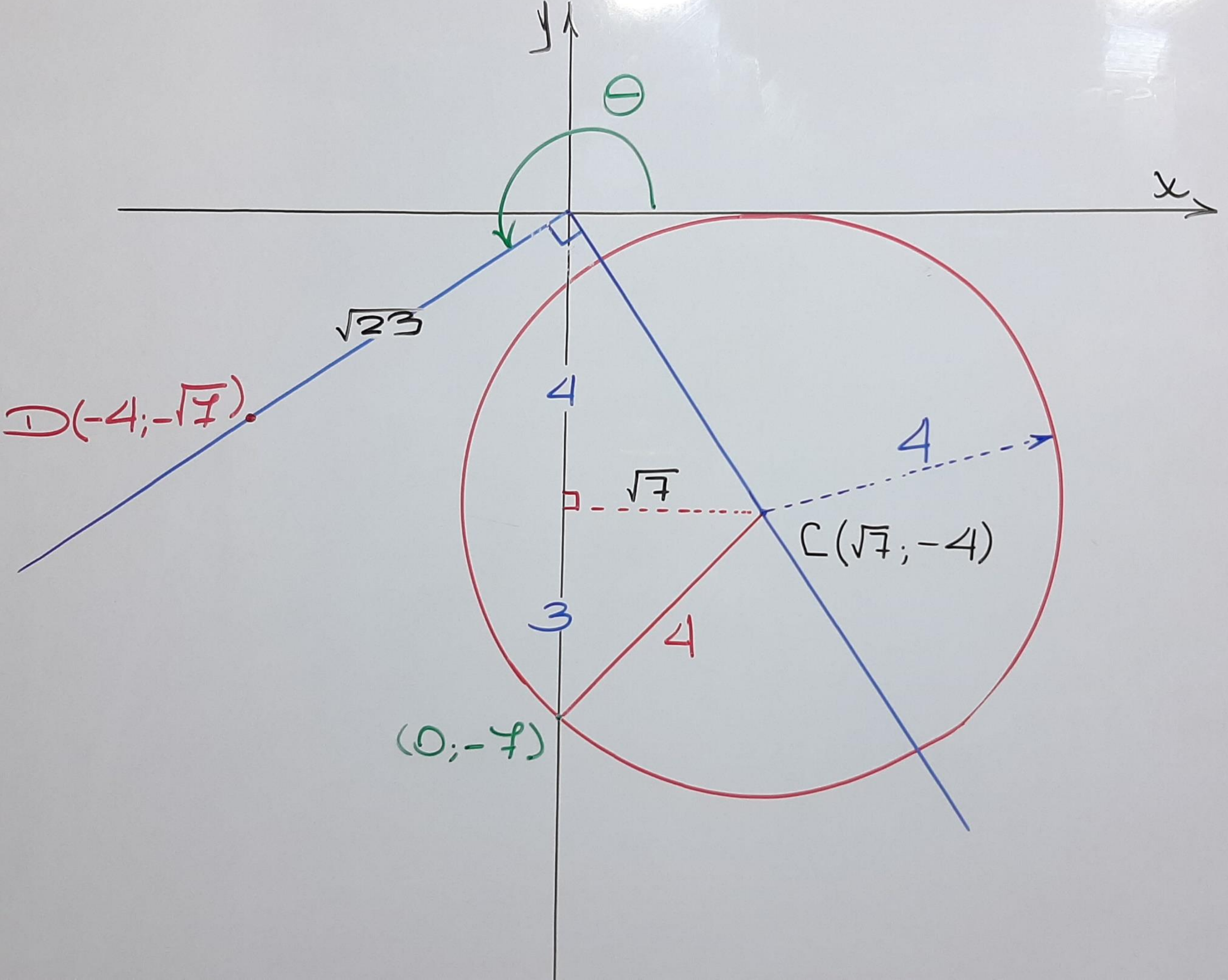
Calcular: $\sqrt{23}\text{Sec}\theta + \sqrt{7}\text{Tan}\theta$



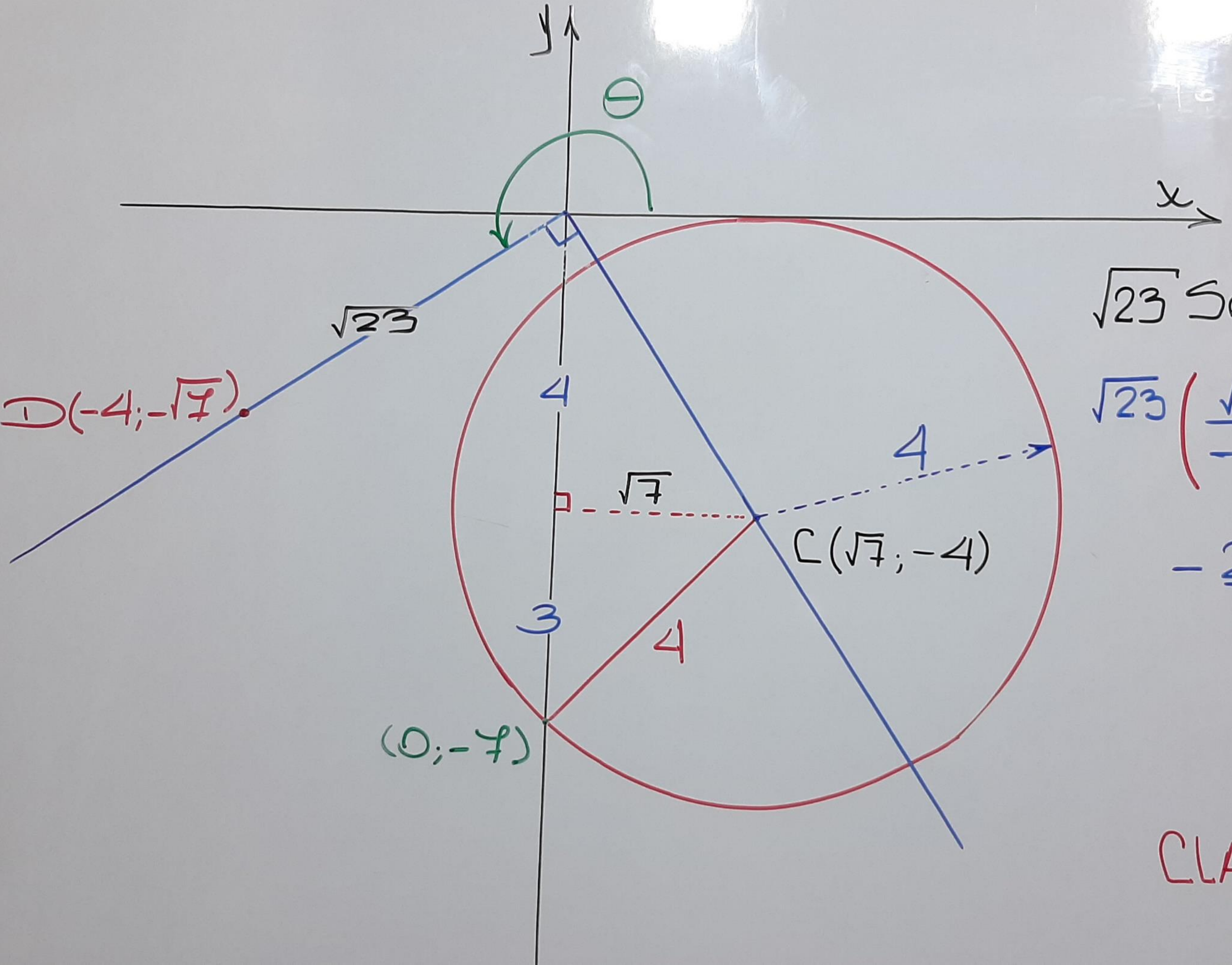
A) -1 B) -2 C) -3
D) -4 E) -5



y
x
θ
4
√7
4
3
4
(0;-7)



y
x
θ
√23
D(-4;-√7)
4
√7
4
C(√7;-4)
3
4
(0;-7)



$$\sqrt{23}\,\mathrm{Sec}\theta + \sqrt{7}\,\mathrm{Tan}\theta$$

$$\sqrt{23}\left(\frac{\sqrt{23}}{-4}\right) + \sqrt{7}\left(\frac{-\sqrt{7}}{-4}\right)$$

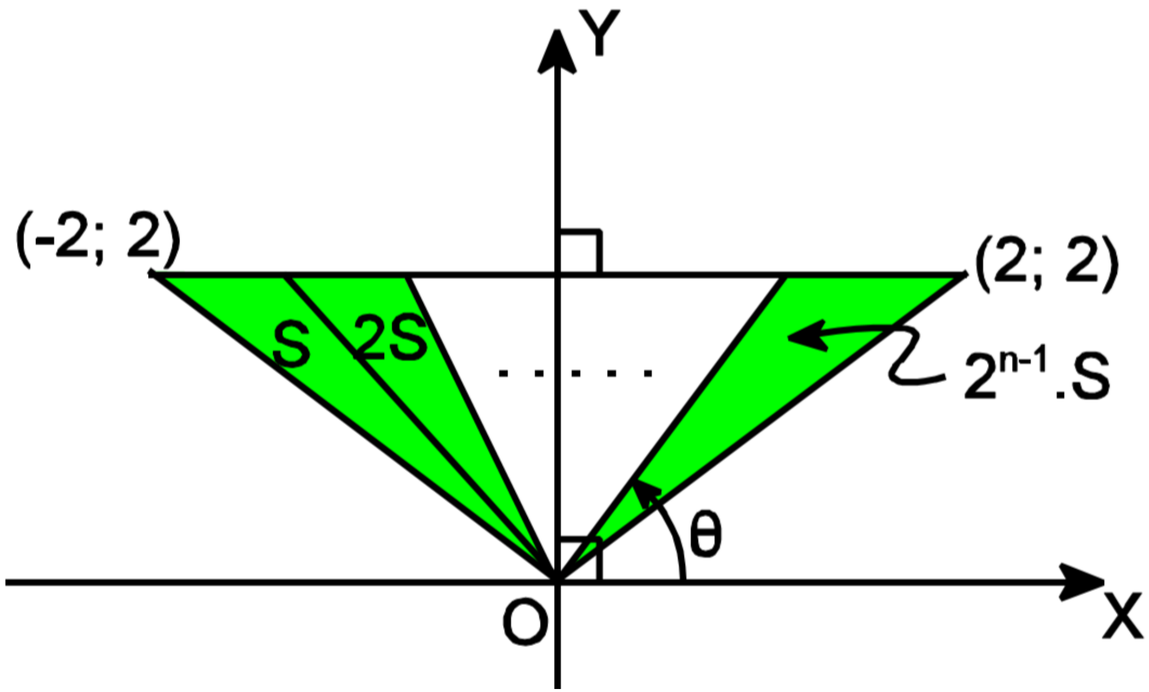
$$-\frac{23}{4} + \frac{7}{4}$$

$$\underline{-4}$$

CLAVE D

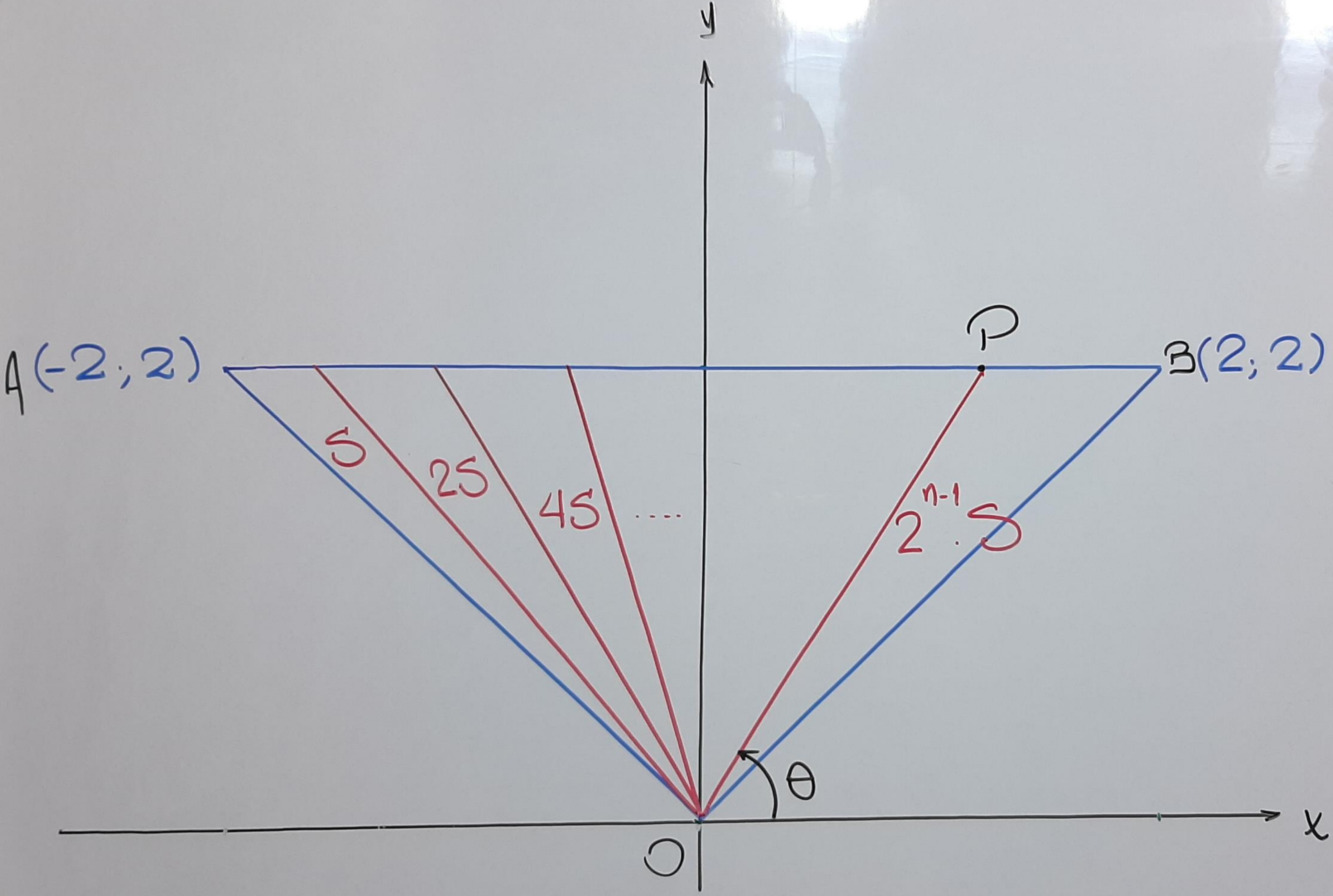
Problema 9:

De la figura mostrada, determine Tanθ

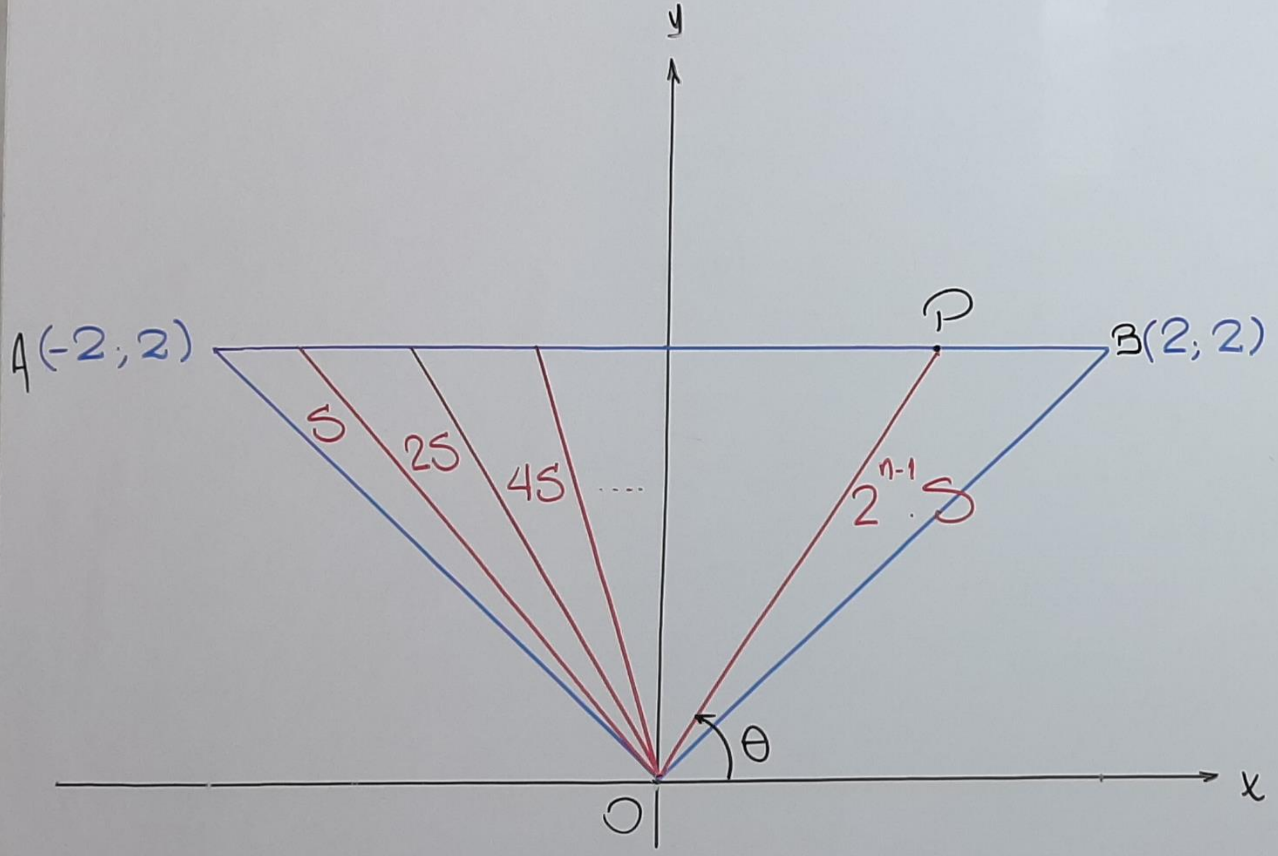


A) $\frac{1}{2^n-1}$ B) $\frac{-1}{2^n-1}$ C) $1-2^n$

D) 2^n-2 E) 2



y
A(-2; 2)
B(2; 2)
P
S
2S
4S
....
$2^{n-1} \cdot S$
θ
O
x

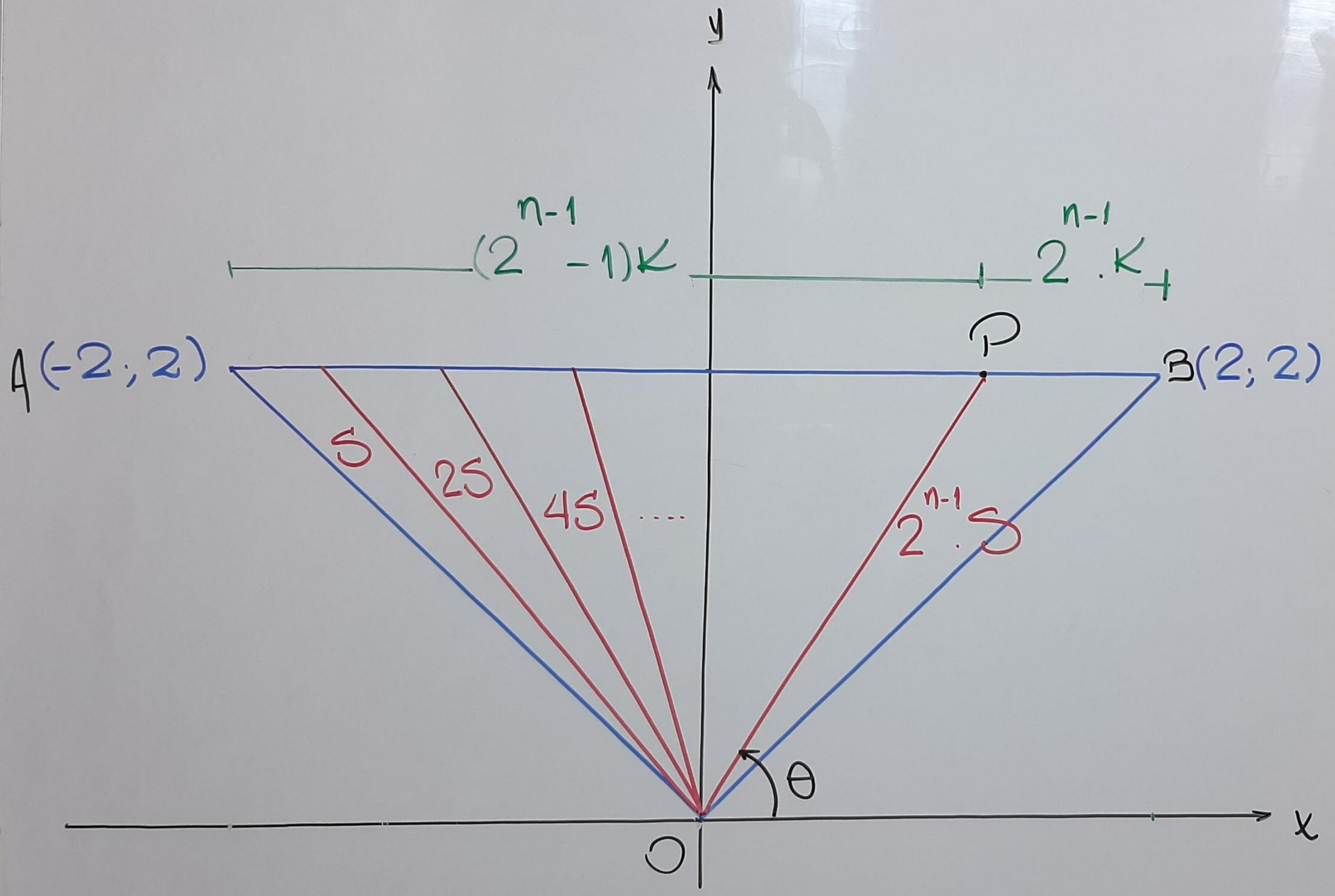


$$S_{\Delta AOP}: S+2S+4S+\ldots+2^{n-2}\cdot S$$

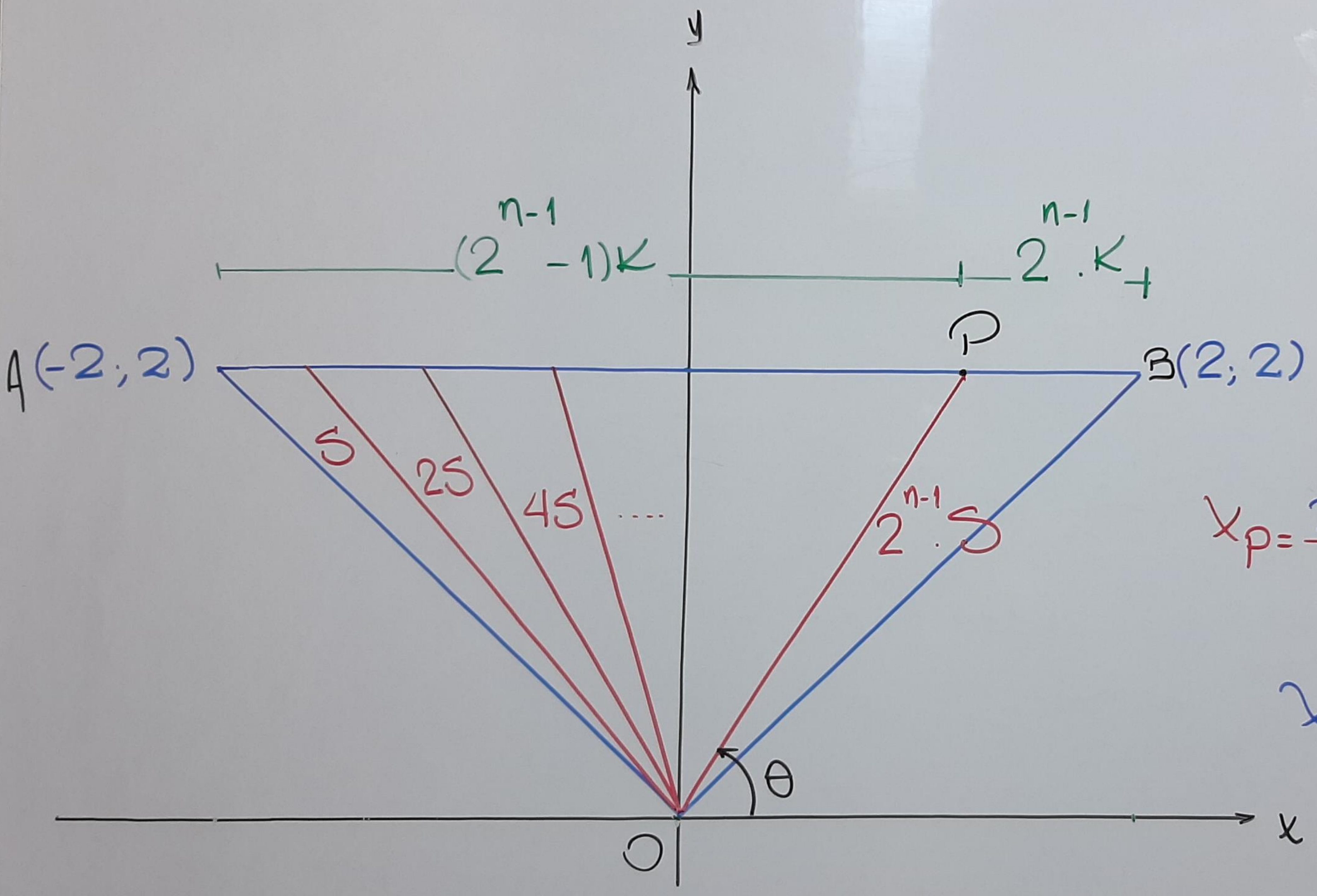
$$S(\underbrace{1+2^1+2^2+\ldots+2^{n-2}})$$

$$S\cdot\frac{1\cdot(2^{n-1}-1)}{(2-1)}$$

$$S_{\Delta AOP}=(2^{n-1}-1)S$$

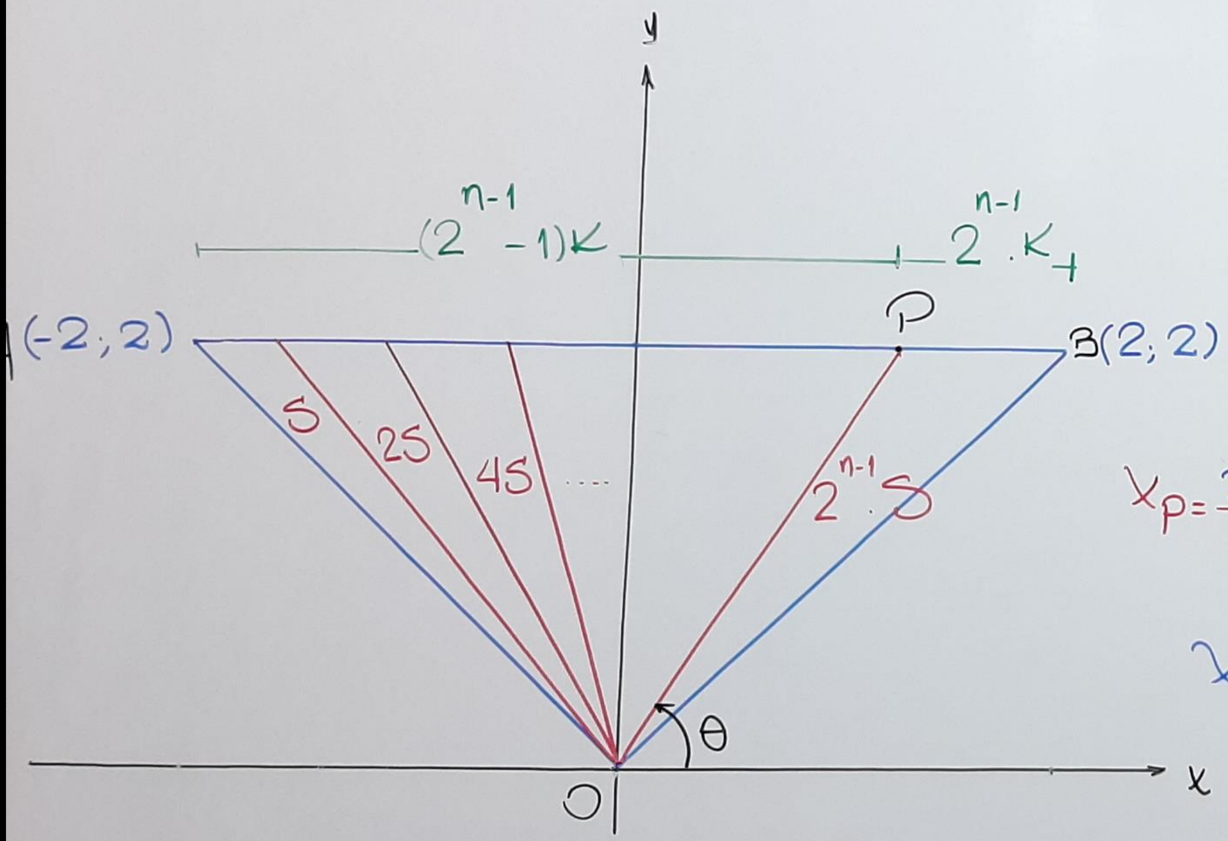


y
$(2^{n-1}-1)K$
$2^{n-1}.K$
P
A(-2; 2)
B(2; 2)
S
2S
4S
....
$2^{n-1}.S$
θ
O
x



$$x_P = \frac{2(2^{n-1}-1)+(-2)(2^{n-1})}{2^{n-1}-1+2^{n-1}}$$

$$x_P = \frac{-2}{2^n-1}$$



$$x_p = \frac{2(2^{n-1}-1) + (-2)(2^{n-1})}{2^{n-1}-1+2^{n-1}}$$

$$x_p = \frac{-2}{2^n-1}$$

$$S_{\Delta AOP}: S+2S+4S+\dots+2^{n-2}S$$

$$S(\underbrace{1+2^1+2^2+\dots+2^{n-2}})$$

$$S \cdot \frac{1.(2^{n-1}-1)}{(2-1)}$$

$$S_{\Delta AOP} = (2^{n-1}-1)S$$

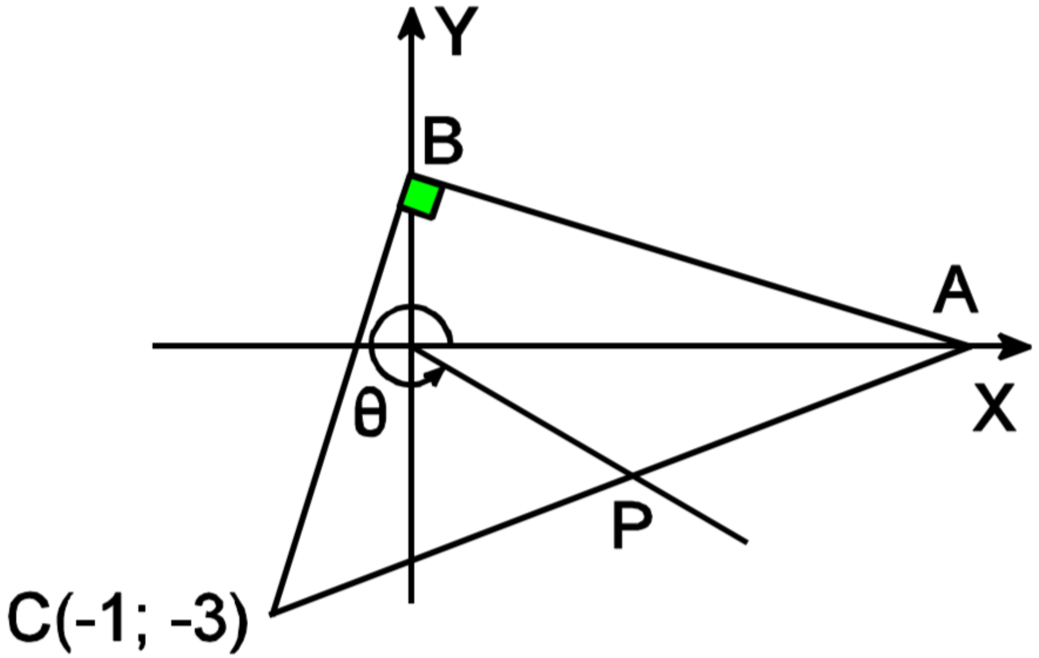
$$\to P\left(\frac{-2}{2^n-1} ; 2\right)$$

$$\tan\theta = \frac{2}{\frac{-2}{2n-1}} \quad \therefore \tan\theta = \underline{1-2^n}$$

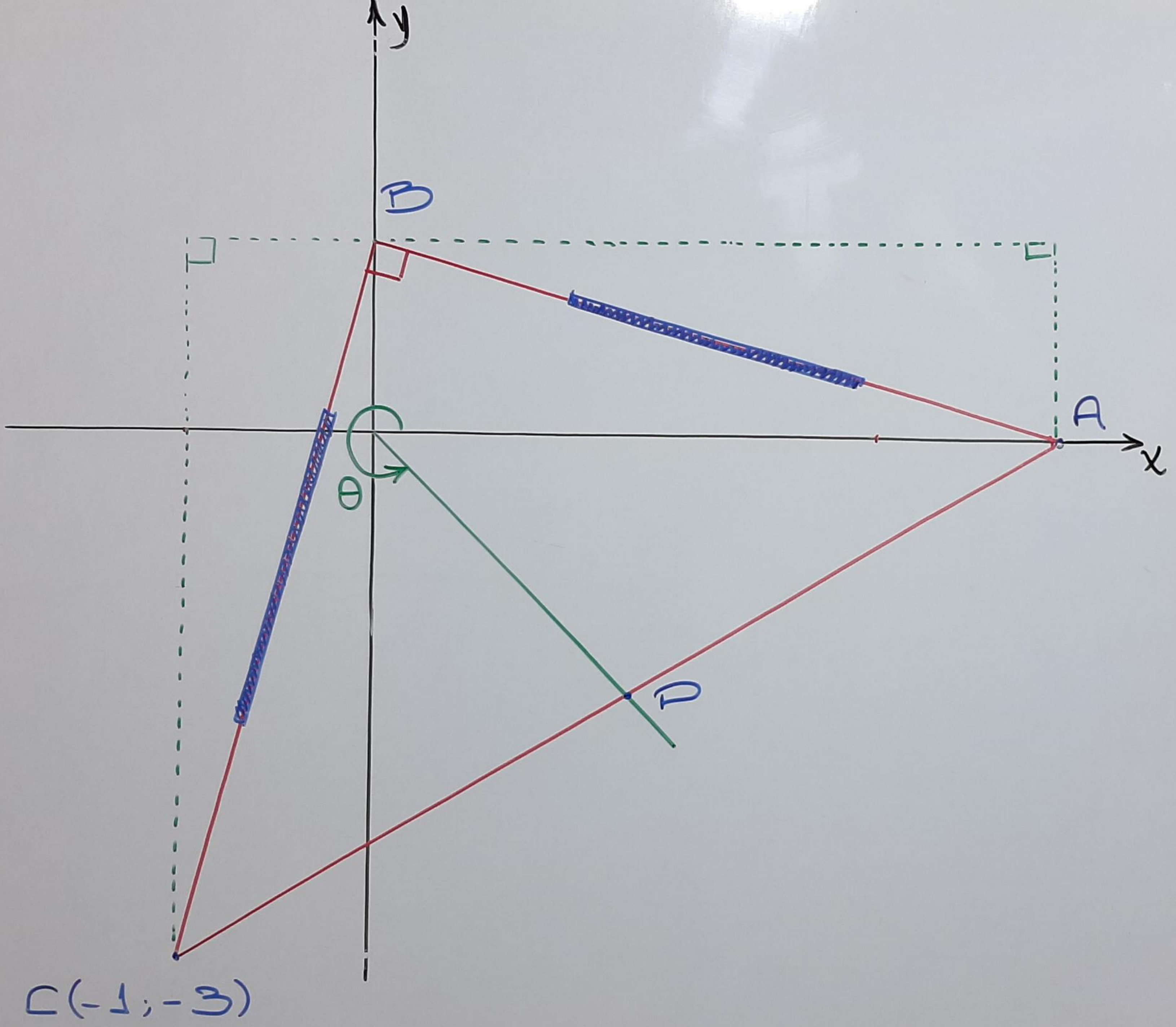
CLAVE C

Problema 10 :

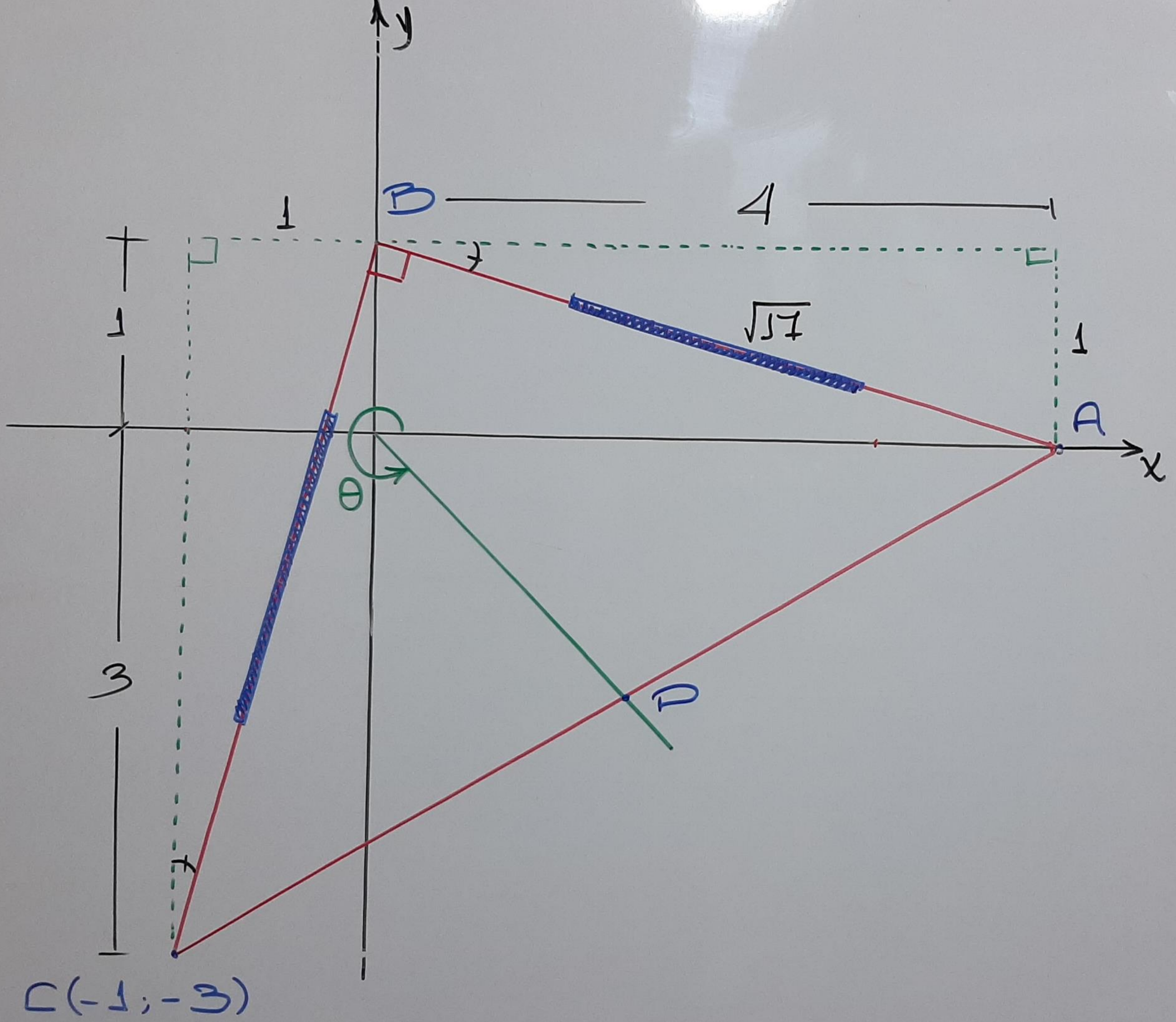
Calcule: Senθ + Cosθ, si el área de la región triangular ABP es $\frac{17}{4}u^2$ y AB=BC.



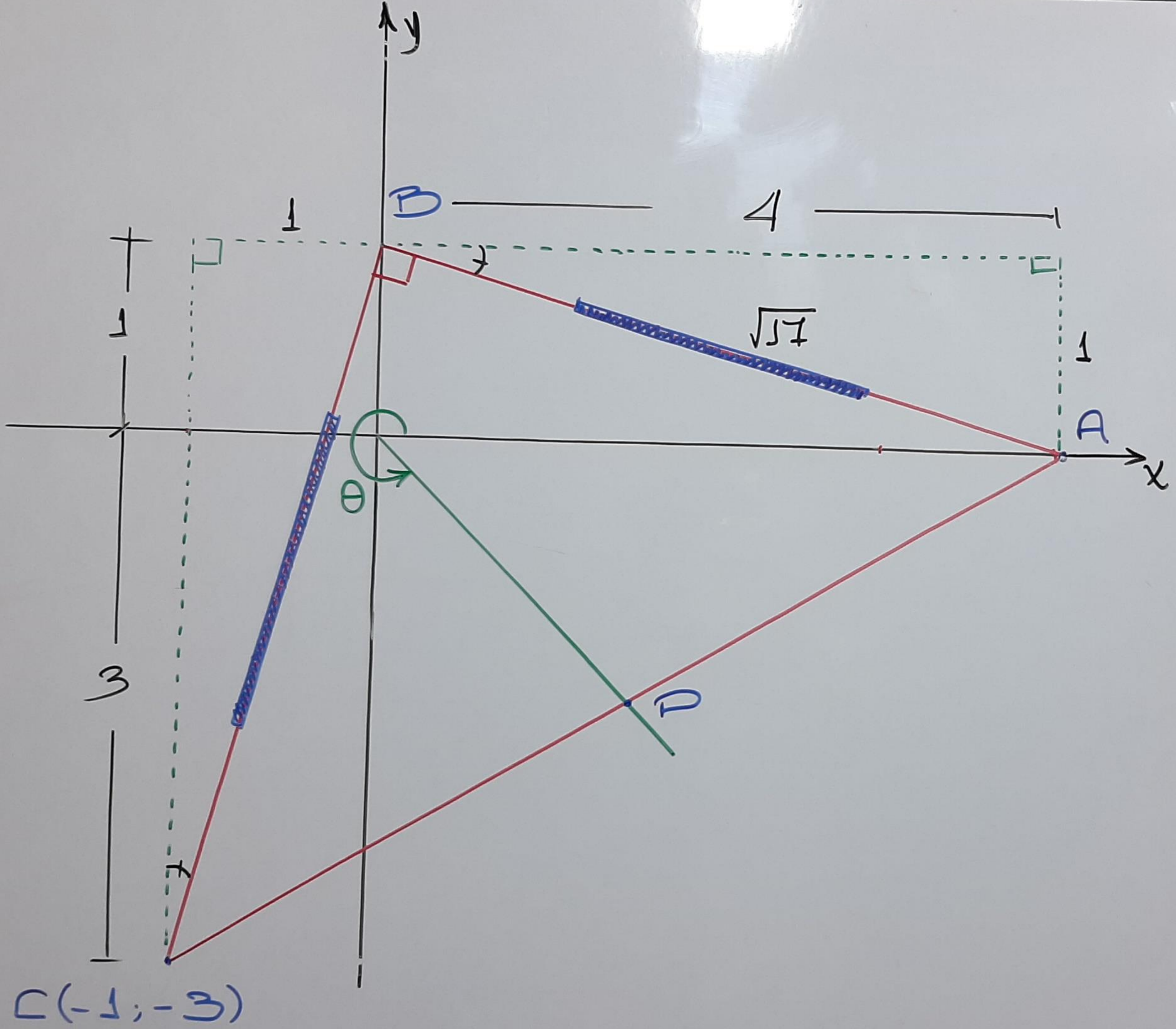
A) $\frac{1}{5}$
B) $-\frac{1}{5}$
C) $-\frac{3}{4}$
D) 0
E) 1



$$S_{\Delta ABP} = \frac{17}{4}$$



$$S_{\Delta ABP} = \frac{17}{4}$$

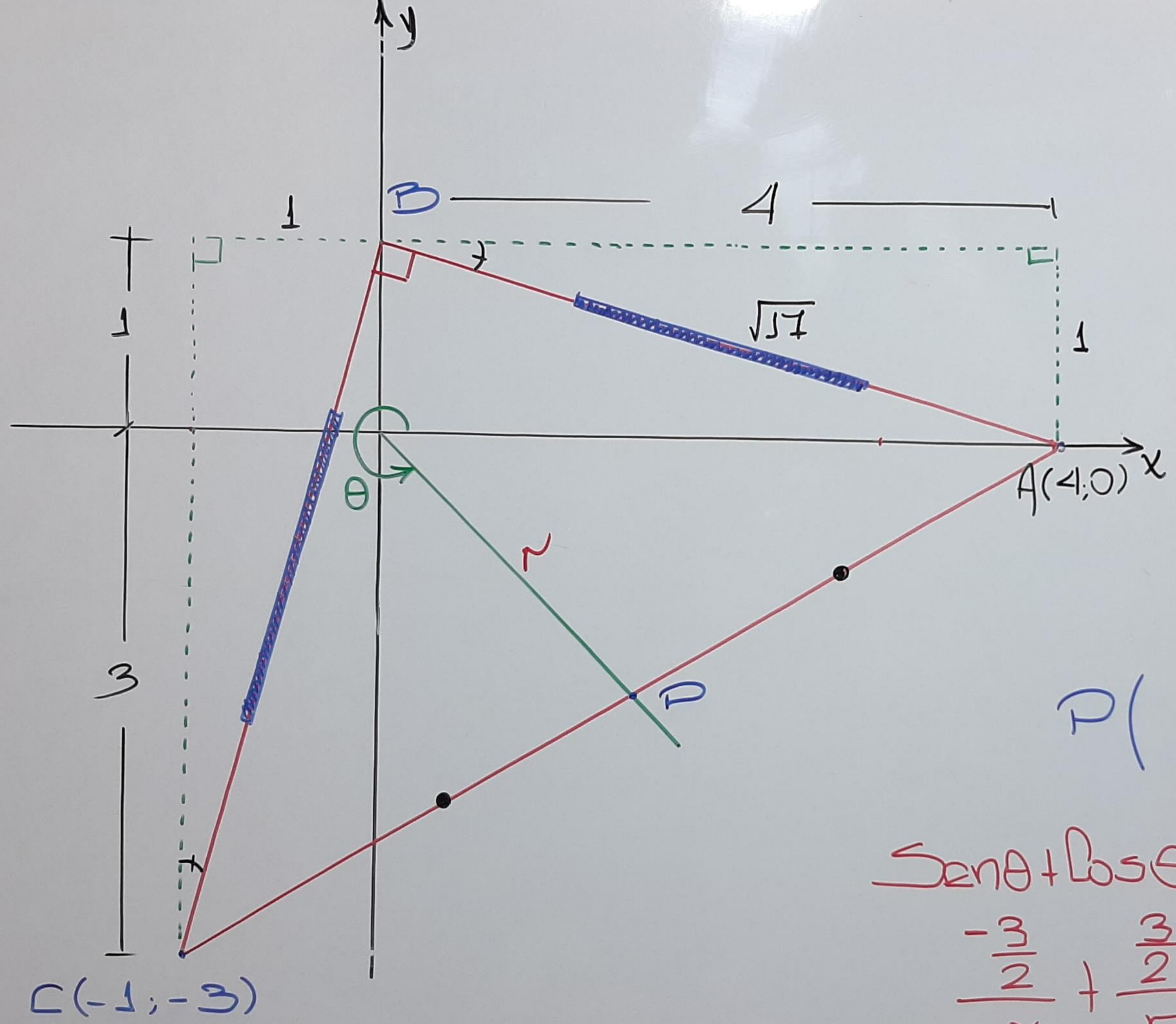


$S_{\Delta ABP} = \frac{17}{4}$

Del gráfico:

$S_{\Delta ABC} = \frac{17}{2}$

2. P: punto medio de $\overline{CA}$



$S_{\Delta ABP} = \dfrac{17}{4}$

Del gráfico:

$S_{\Delta ABC} = \dfrac{17}{2}$

2. P: punto medio de $\overline{CA}$

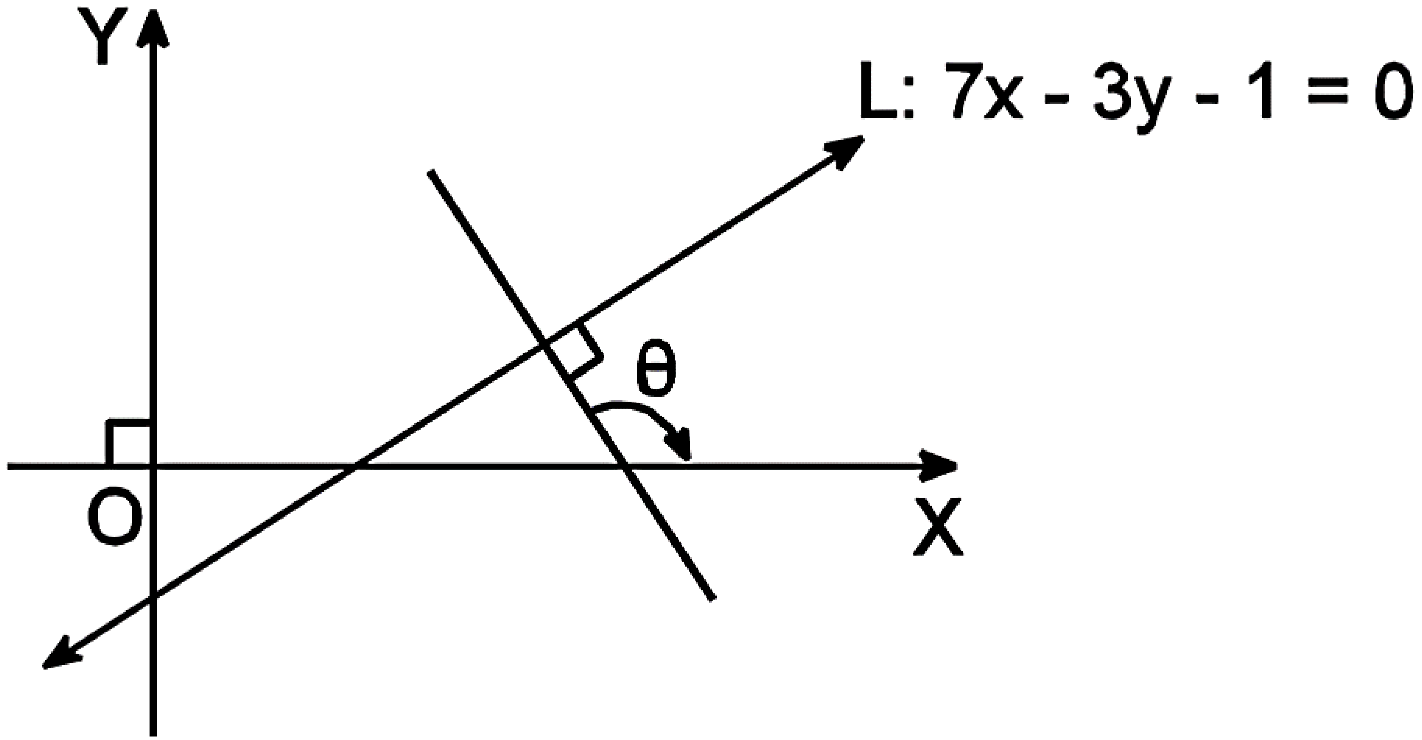
$P\left(\dfrac{3}{2} ; -\dfrac{3}{2}\right)$

$\operatorname{Sen}\theta + \operatorname{Cos}\theta$

$\dfrac{-\frac{3}{2}}{r} + \dfrac{\frac{3}{2}}{r} = 0$

Problema 11:

En la figura mostrad, calcule: $K = \text{Tan}\theta + \sqrt{58}\text{Sec}\theta$

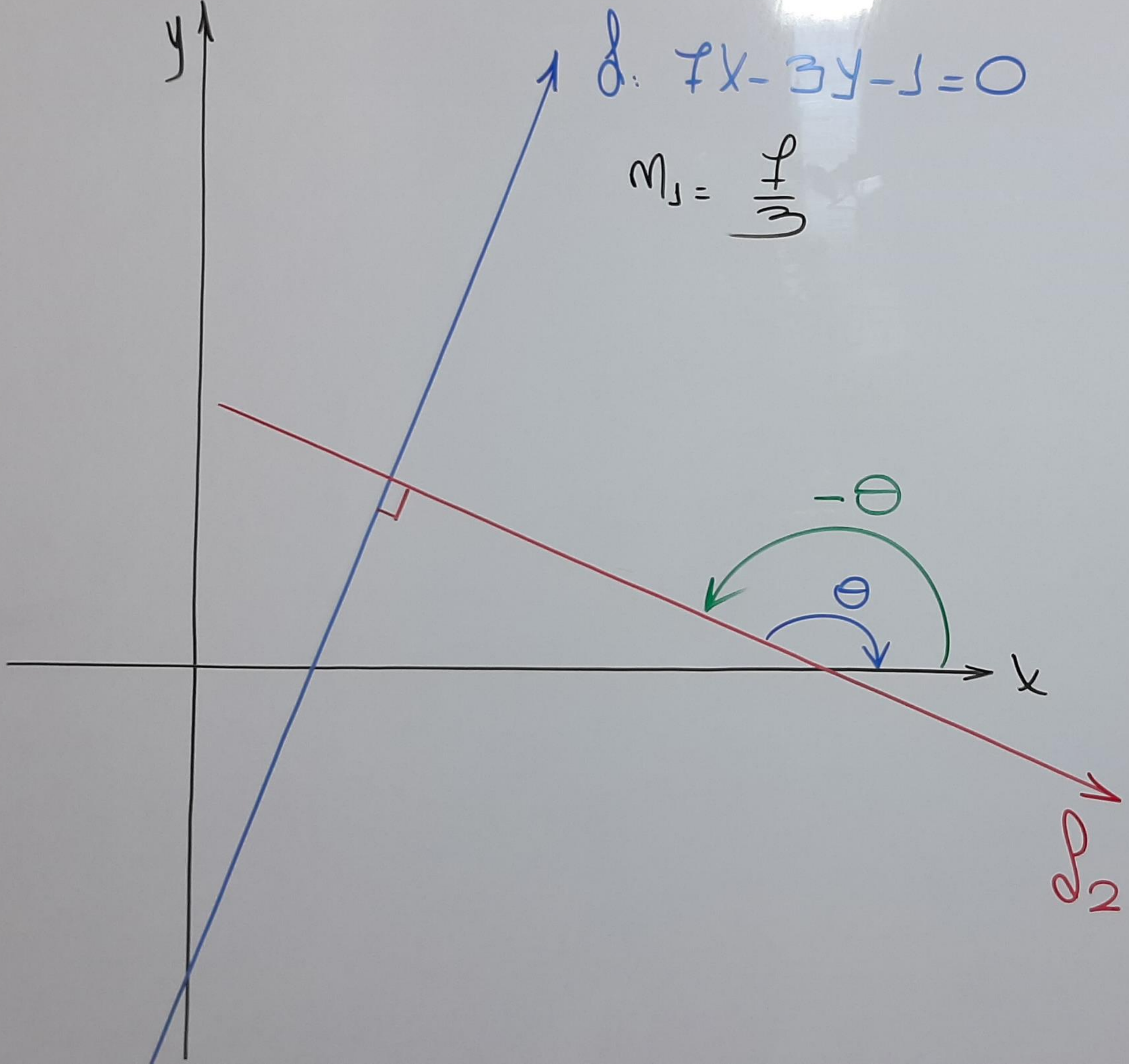


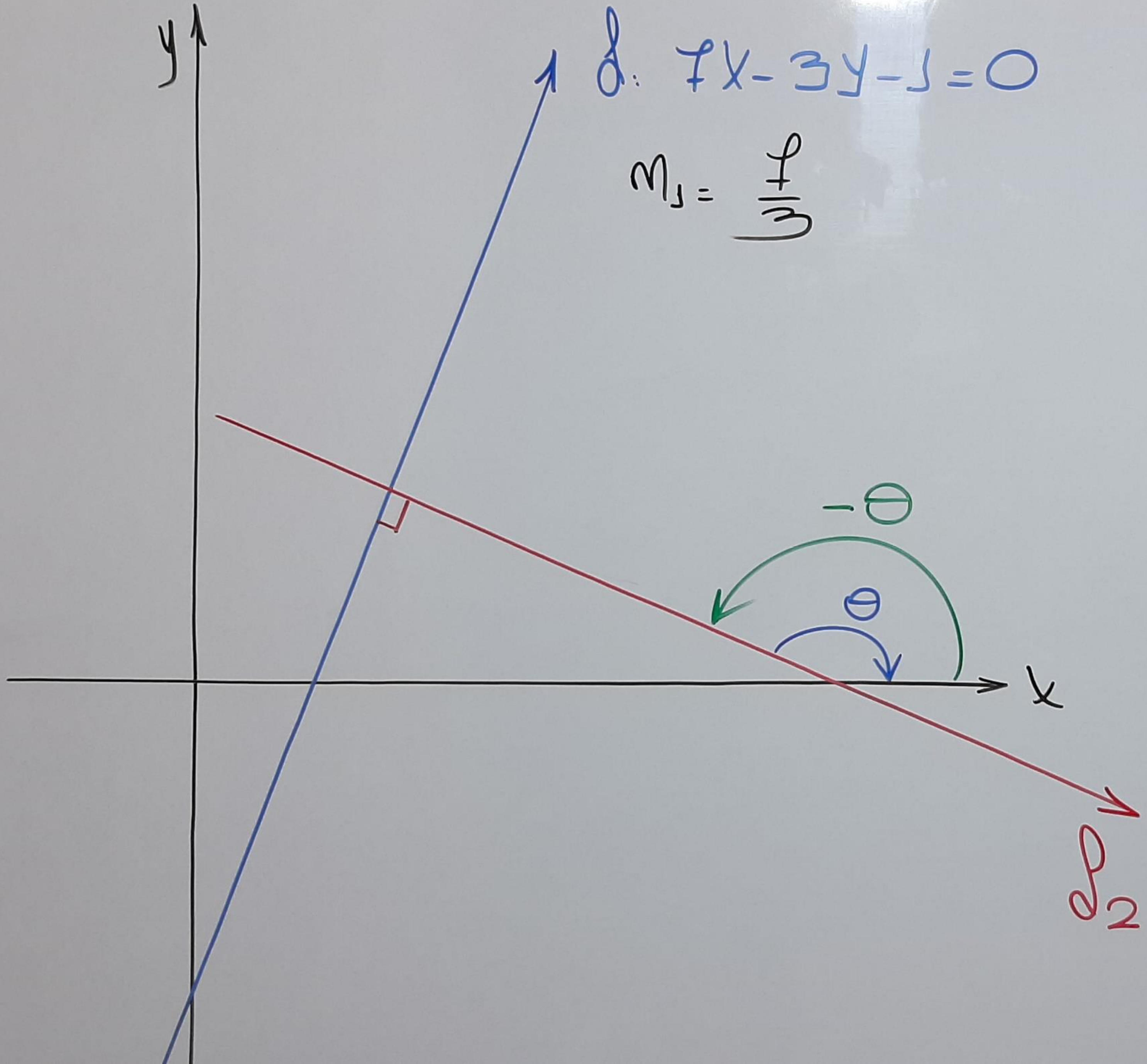
A) $-55/7$ B) $1/7$ C) $-\sqrt{5}/7$

D) -55 E) $55/7$

$\ell: 7x - 3y - 1 = 0$

$m_1 = \frac{7}{3}$





$$\ell : 7x - 3y - 1 = 0$$

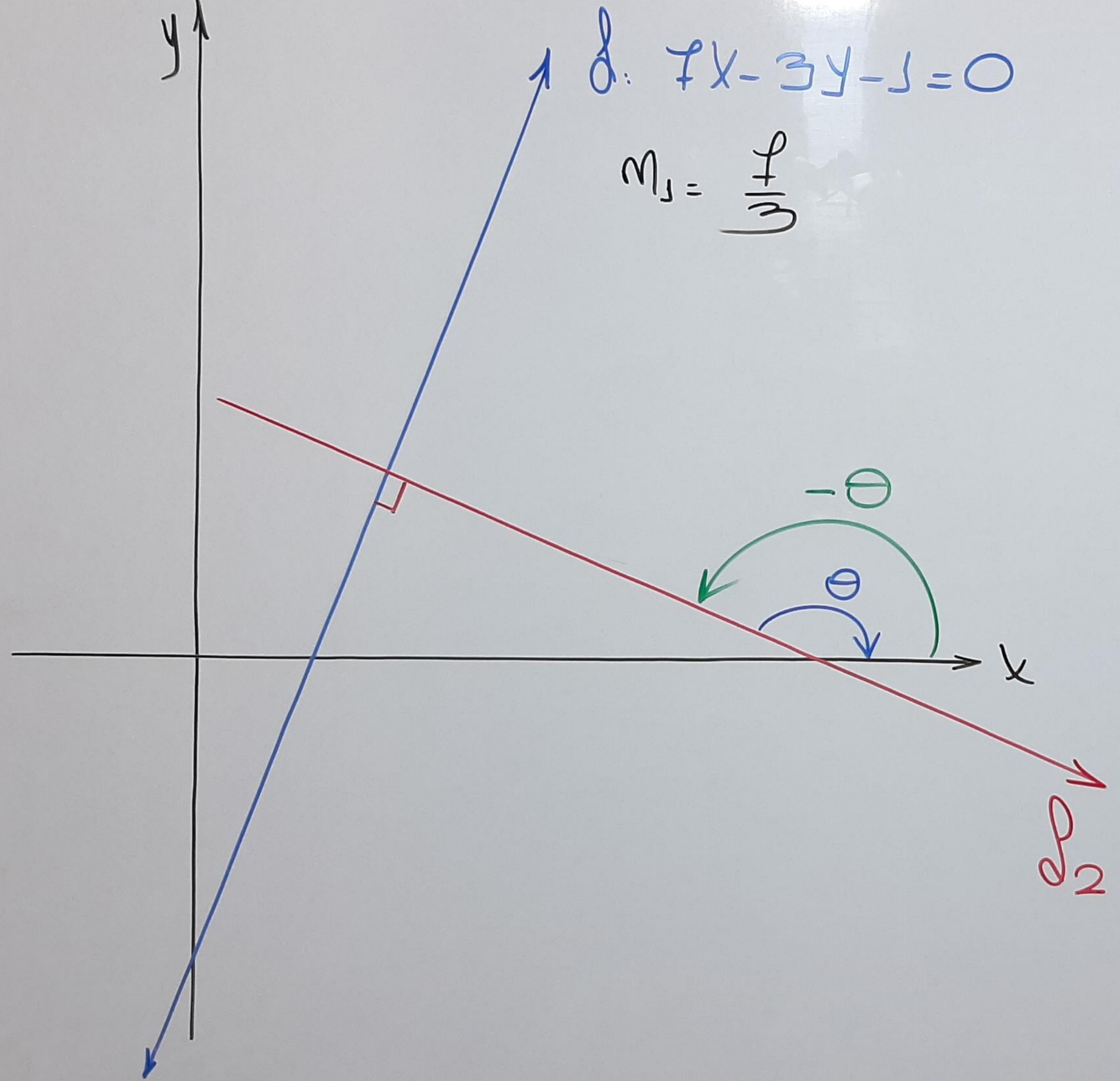
$$m_1 = \frac{7}{3}$$

$$\hookrightarrow \underbrace{m_2} = -\frac{3}{7}$$

$$\tan(\underbrace{-\theta}_{\mathrm{IIC}}) = -\frac{3}{7}$$

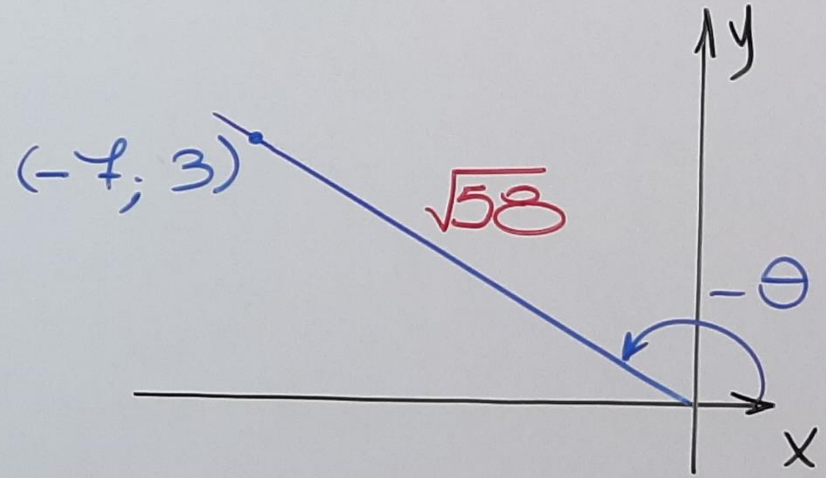
$\mathcal{L}_1 : 7x - 3y - 1 = 0$

$m_1 = \dfrac{7}{3}$



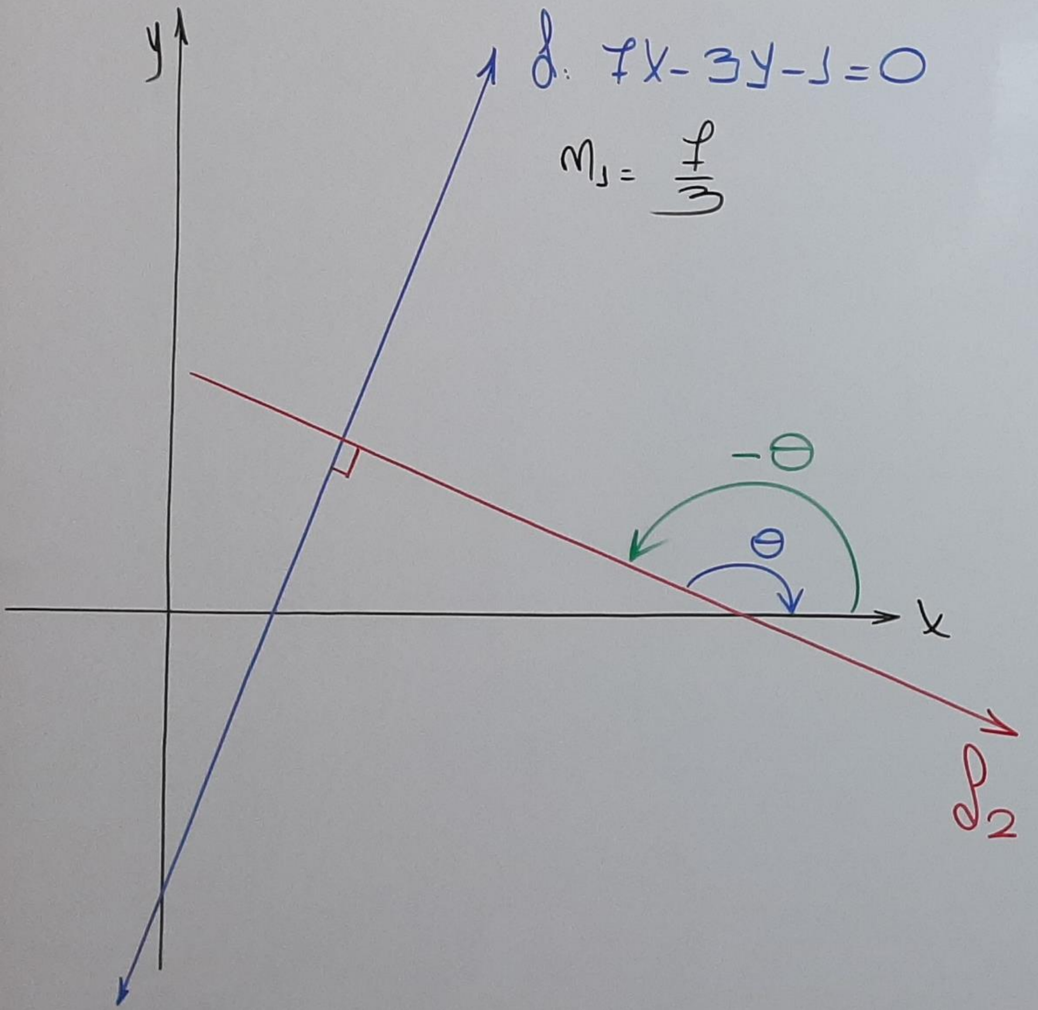
$\underbrace{\perp\ m_2}_{} = -\dfrac{3}{7}$

$\tan(\underbrace{-\theta}_{\text{IIC}}) = -\dfrac{3}{7}$



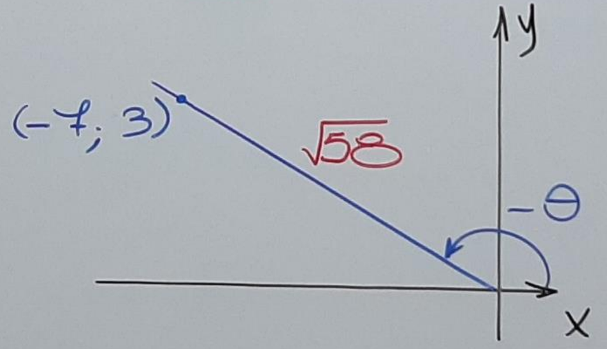
$\mathcal{L}_1: 7x - 3y - 1 = 0$

$m_1 = \frac{7}{3}$



$\perp \quad \underbrace{m_2}_{} = -\frac{3}{7}$

$\tan(\underbrace{-\theta}_{\text{IIC}}) = -\frac{3}{7}$



Calcular:

$K = \operatorname{Tan}\theta + \sqrt{58}\operatorname{Sec}\theta$

$K = -\operatorname{Tan}(-\theta) + \sqrt{58} \cdot \operatorname{Sec}(-\theta)$

$K = -\left(-\frac{3}{7}\right) + \sqrt{58}\left(\frac{\sqrt{58}}{-7}\right)$

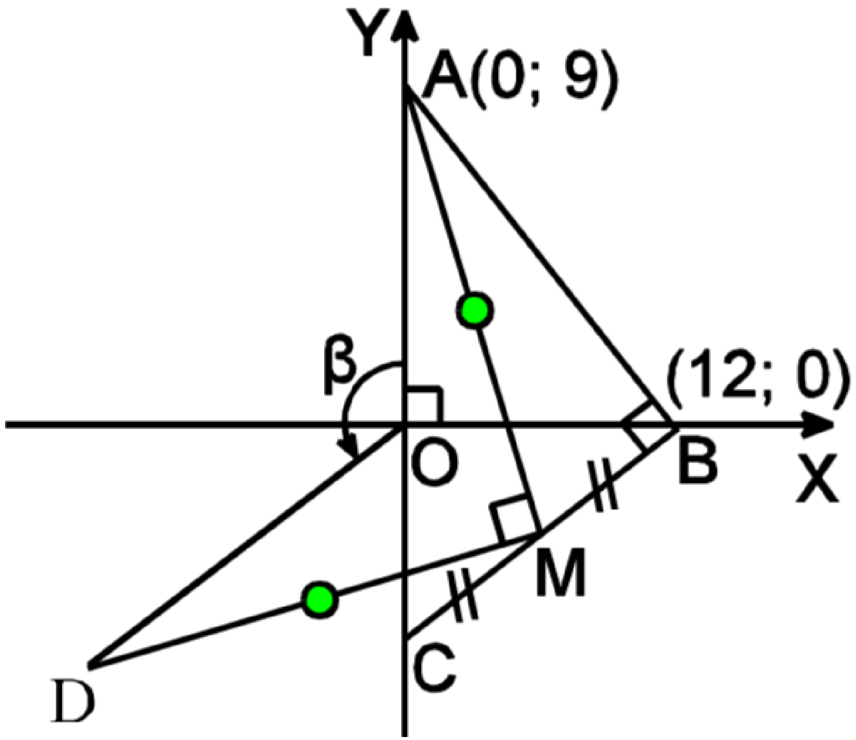
$K = \frac{3}{7} - \frac{58}{7}$

$K = -\frac{55}{7}$

CLAVE A

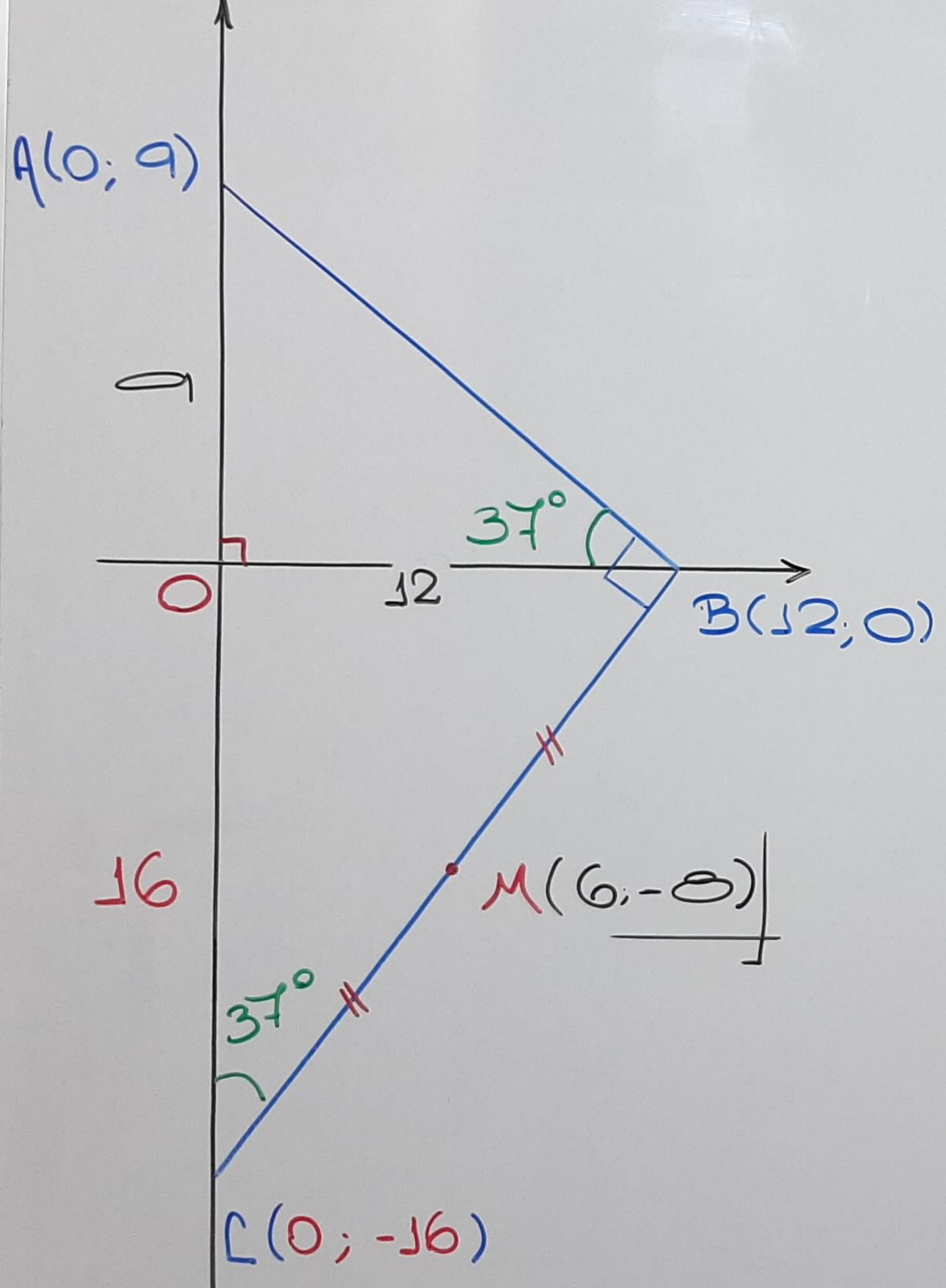
Problema 12:

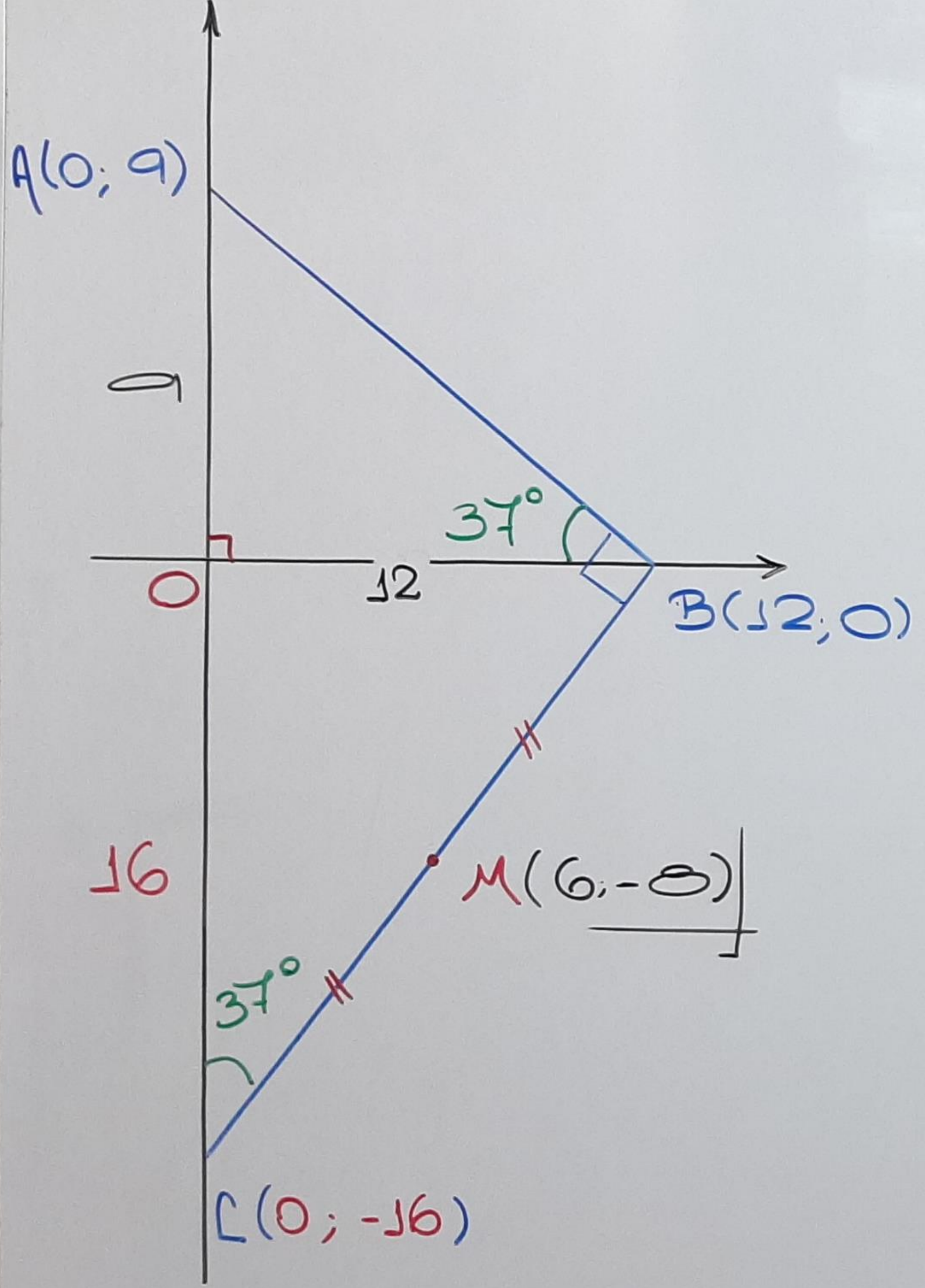
Del grafico, calcule Tanβ, si: AM=MD.

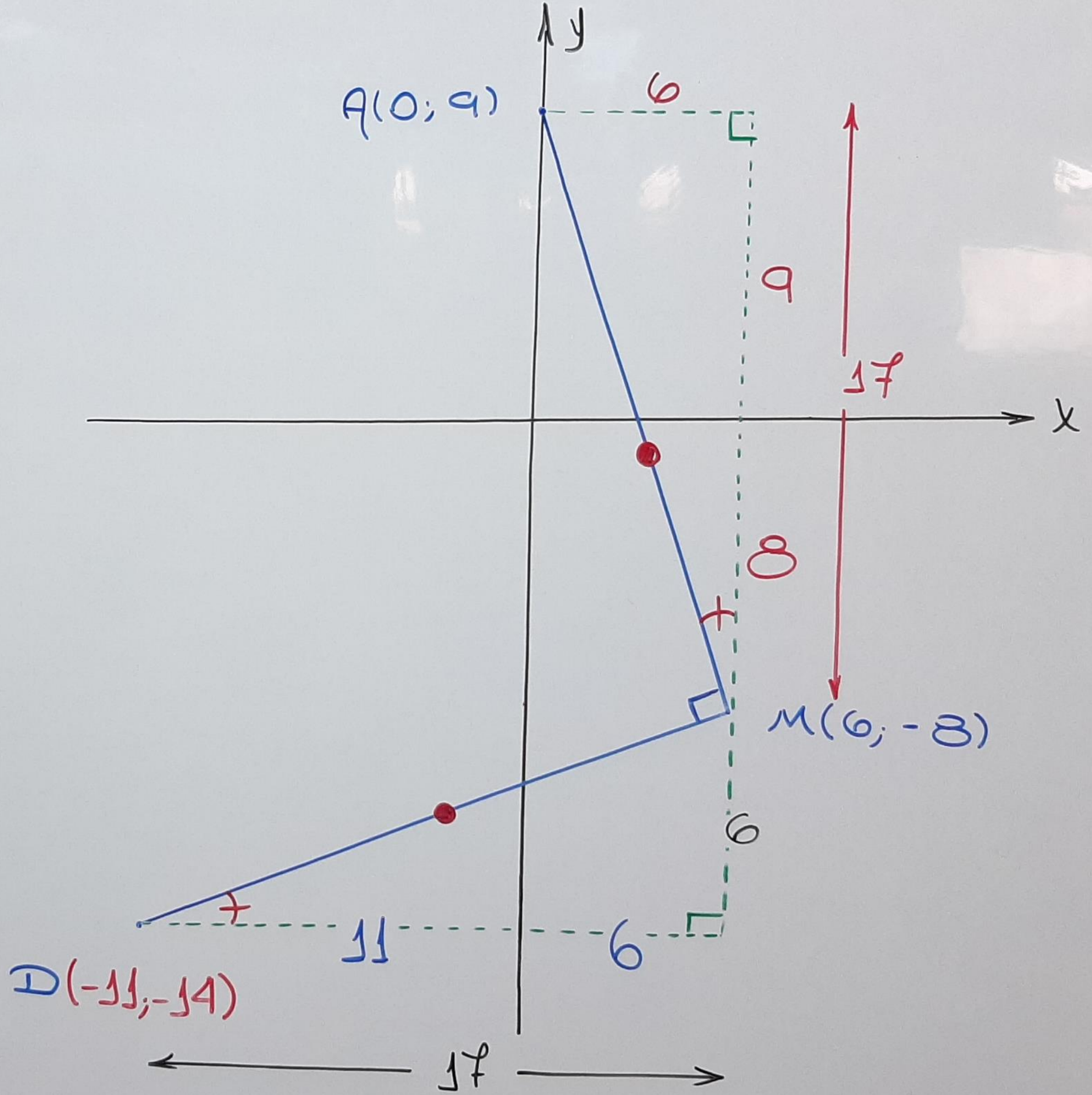


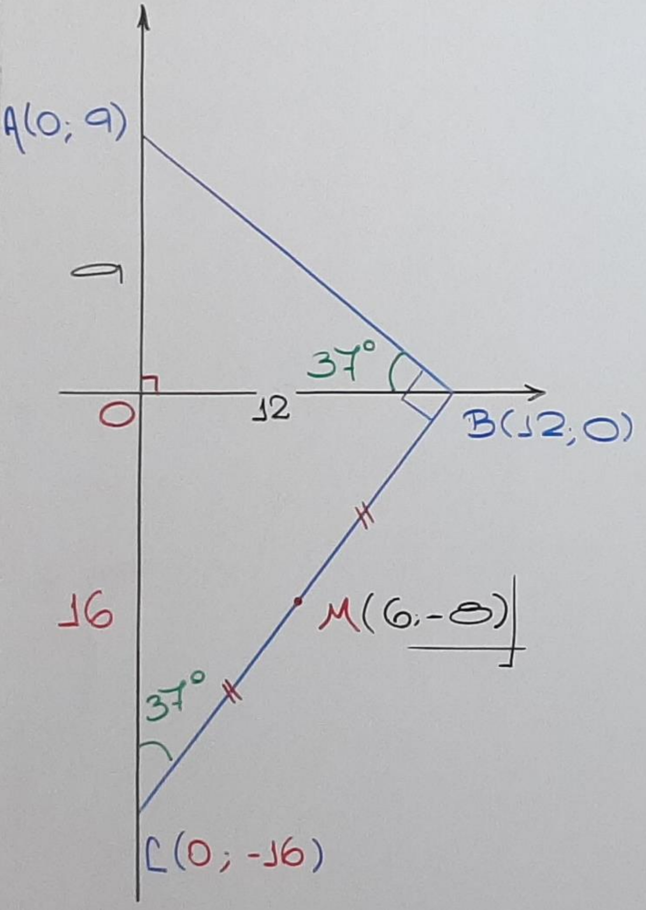
A) $-11/4$ B) $-10/9$ C) $-13/14$

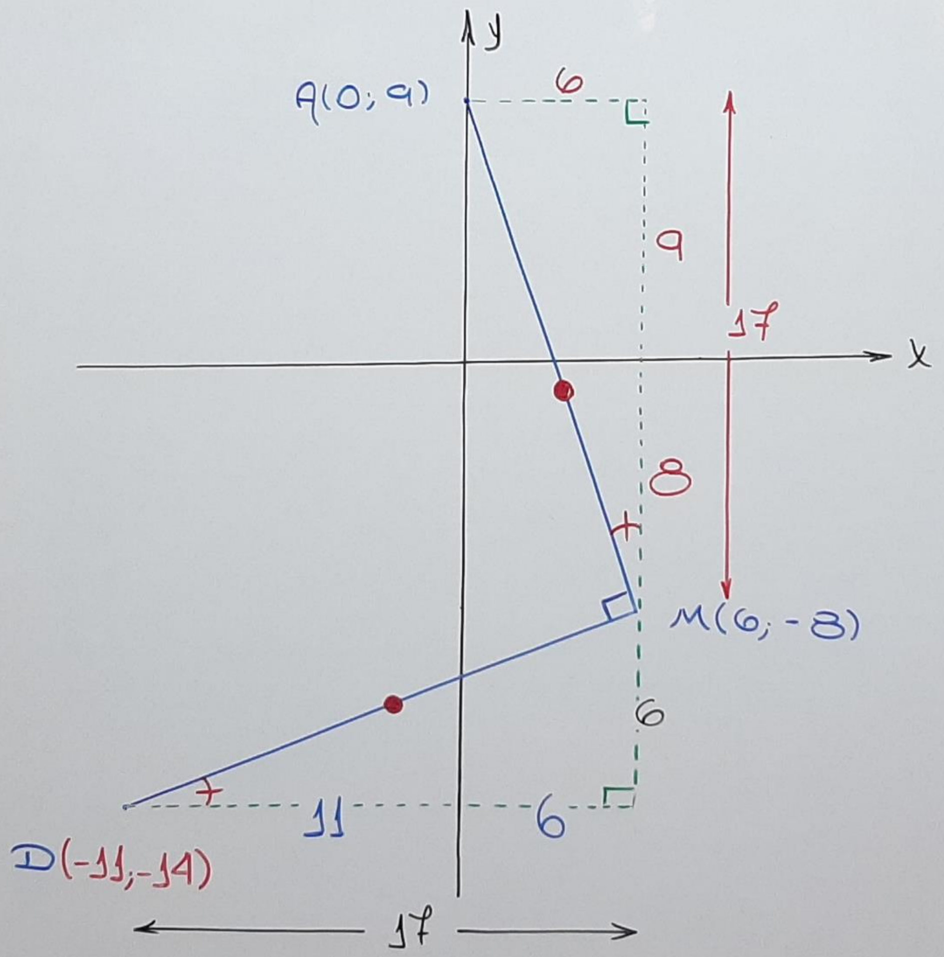
D) $-5/8$ E) $-7/9$


A(0; 9)
9
37°
O
12
B(12; 0)
16
M(6; -8)
37°
C(0; -16)

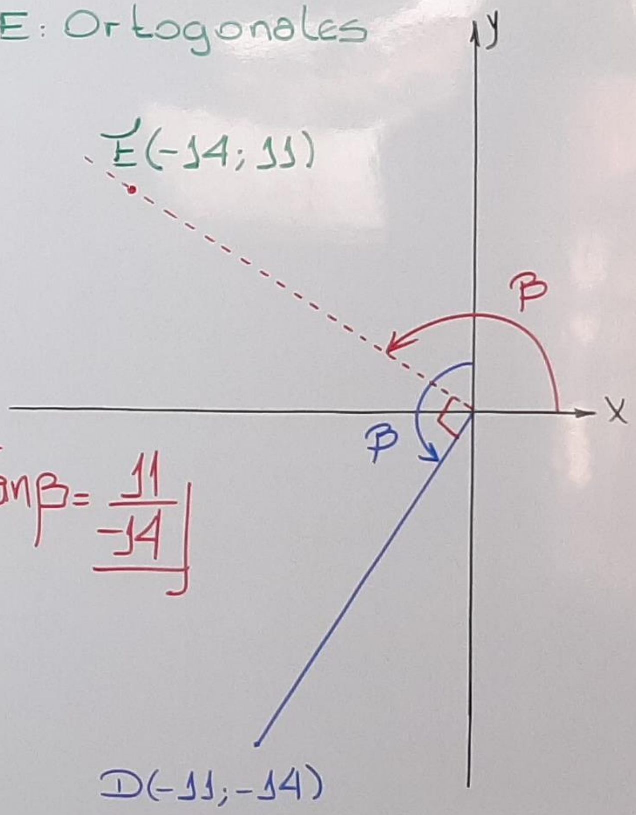

A(0; 9)
9
37°
O
12
B(12;0)
16
M(6;-8)
37°
C(0; -16)


y
A(0; 9)
6
9
17
x
8
M(6; -8)
6
11
6
D(-11;-14)
17





D y E: Ortogonales



$\therefore \tan\beta = \boxed{\dfrac{11}{-14}}$

CLAVE "A"

Problema 13:

Hallar el valor de: $\text{Tan}n\pi + \text{Cos}n\pi + \text{Sen}n\pi, n\epsilon\mathbb{Z}$

A) 1 B) -1 C) 1^n

D) $(-1)^n$ E) 0

$$\underbrace{\text{Tan}(n\pi)}_{0} + \text{Cos}(n\pi) + \underbrace{\text{Sen}(n\pi)}_{0} \; ; n \in \mathbb{Z}$$

✓ $1 \rightarrow n$: par

✓ $-1 \rightarrow n$: impar

$$\underbrace{\tan(n\pi)}_{0} + \cos(n\pi) + \underbrace{\operatorname{Sen}(n\pi)}_{0} \; ; n \in \mathbb{Z}$$

✓ $1 \rightarrow n$: par

✓ $-1 \rightarrow n$: impar

$$\therefore \boxed{(-1)^n}$$

CLAVE D

Problema 14:

Reducir la expresión:

$$\frac{\mathrm{Sec}\left(25\frac{\pi}{2}-x\right)\mathrm{Sen}\left(7\frac{\pi}{2}+x\right)}{\mathrm{Sec}(15\pi-x).\mathrm{Cos}(5\pi-x)}$$

A) 1

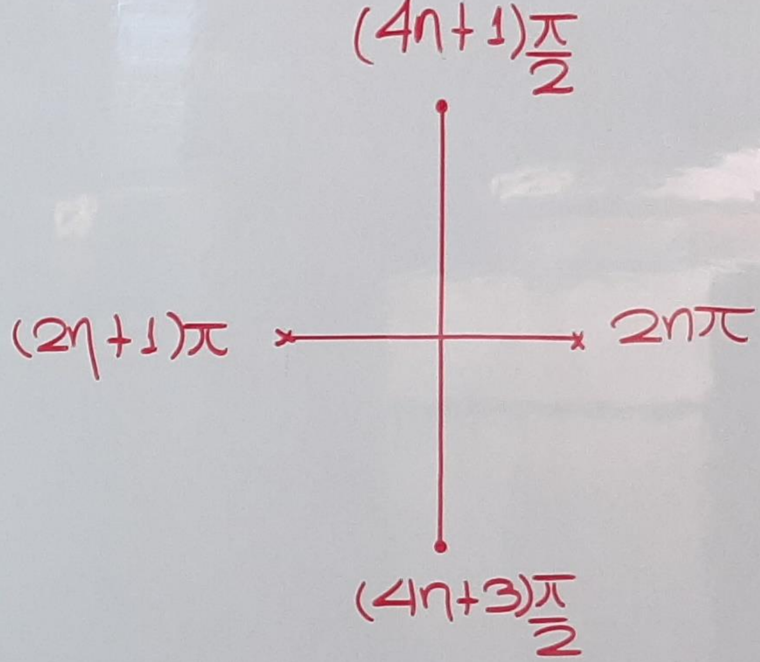
B) -1

C) $-\mathrm{Tan}x$

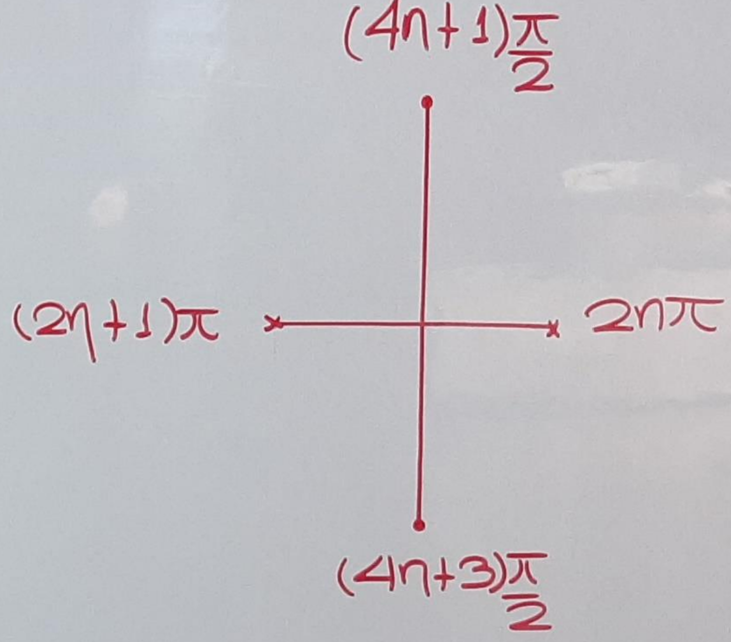
D) $\mathrm{Cot}x$

E) $-\mathrm{Cot}x$

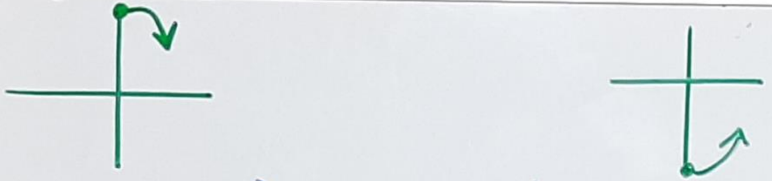
$$\frac{\operatorname{Sec}\left(25\frac{\pi}{2}-x\right)\cdot \operatorname{Sen}\left(7\frac{\pi}{2}+x\right)}{\operatorname{Sec}(15\pi-x)\cdot \operatorname{Cos}(5\pi-x)}$$



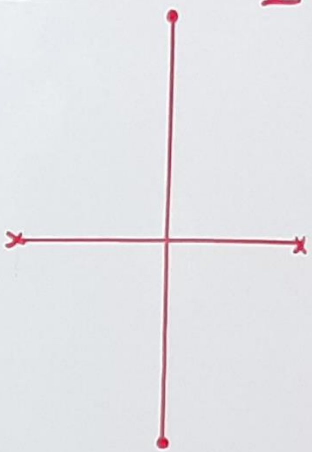
$$\frac{\operatorname{Sec}\left(\frac{25\pi}{2}-x\right)\cdot \operatorname{Sen}\left(\frac{7\pi}{2}+x\right)}{\operatorname{Sec}(15\pi-x)\cdot \operatorname{Cos}(5\pi-x)}$$



$$\frac{(+\operatorname{Csc}x)\cdot(-\operatorname{Cos}x)}{(-\operatorname{Sec}x)\cdot(-\operatorname{Cos}x)}$$



$$\frac{\operatorname{Sec}\left(\frac{25\pi}{2}-x\right)\cdot\operatorname{Sen}\left(\frac{7\pi}{2}+x\right)}{\operatorname{Sec}(15\pi-x)\cdot\operatorname{Cos}(5\pi-x)}$$



$$\frac{(+\operatorname{Csc}x)\cdot(-\operatorname{Cos}x)}{(-\underbrace{\operatorname{Sec}x)\cdot(-\operatorname{Cos}x}_{1})}$$

$$-\operatorname{Cot}x$$

CLAVE E

Problema 15:

Si: $a+b=90°$, hallar el valor de:

$$\frac{3\mathrm{Sen}(2a+b)+4\mathrm{Cos}a}{4\mathrm{Cos}(2b+a)+3\mathrm{Sen}b}$$

A) $3\mathrm{Tan}a$ B) $-3\mathrm{Tan}a$ C) $-3\mathrm{Cot}a$

D) $3\mathrm{Cot}a$ E) -7

$a + b = 90° \rightarrow b = 90° - a$

$$\frac{3\,Sen(2a+b) + 4\,Cos\,a}{4 \cdot Cos(2b+a) + 3\,Sen\,b}$$

$a + b = 90° \rightarrow b = 90° - a$

$$\frac{3\,Sen(2a+b) + 4\,Cos\,a}{4 \cdot Cos(2b+a) + 3\,Sen\,b}$$

$$\frac{3 \cdot Sen(90° + a) + 4\,Cos\,a}{4 \cdot Cos(180° - a) + 3\,Sen(90° - a)}$$

$a + b = 90° \to b = 90° - a$

$$\frac{3\,\mathrm{Sen}(2a+b) + 4\,\mathrm{Cos}\,a}{4.\,\mathrm{Cos}(2b+a) + 3\,\mathrm{Sen}\,b}$$

$$\frac{3.\,\mathrm{Sen}(90° + a) + 4\,\mathrm{Cos}\,a}{4.\,\mathrm{Cos}(180° - a) + 3\,\mathrm{Sen}(90° - a)}$$

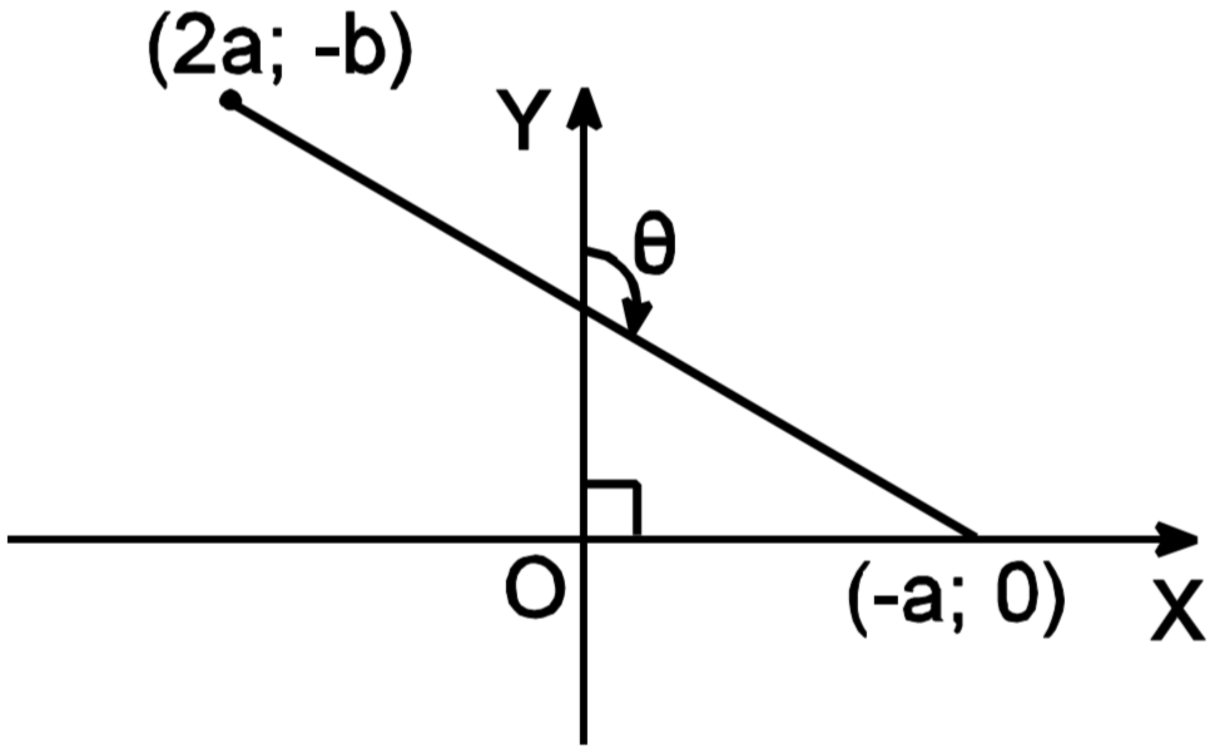
$$\frac{3(\mathrm{Cos}\,a) + 4\,\mathrm{Cos}\,a}{4(-\mathrm{Cos}\,a) + 3(\mathrm{Cos}\,a)}$$

$$-7$$

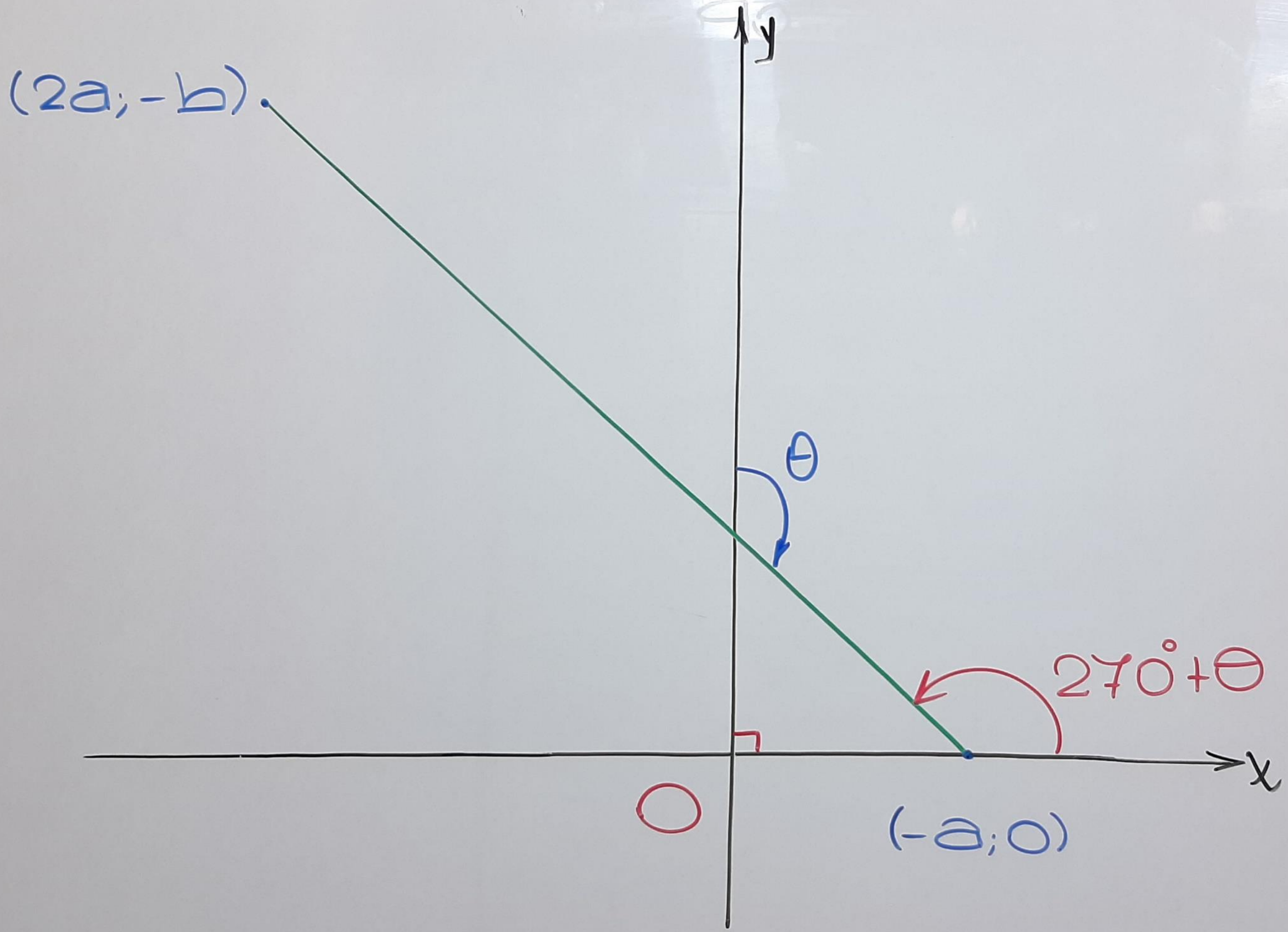
CLAVE E

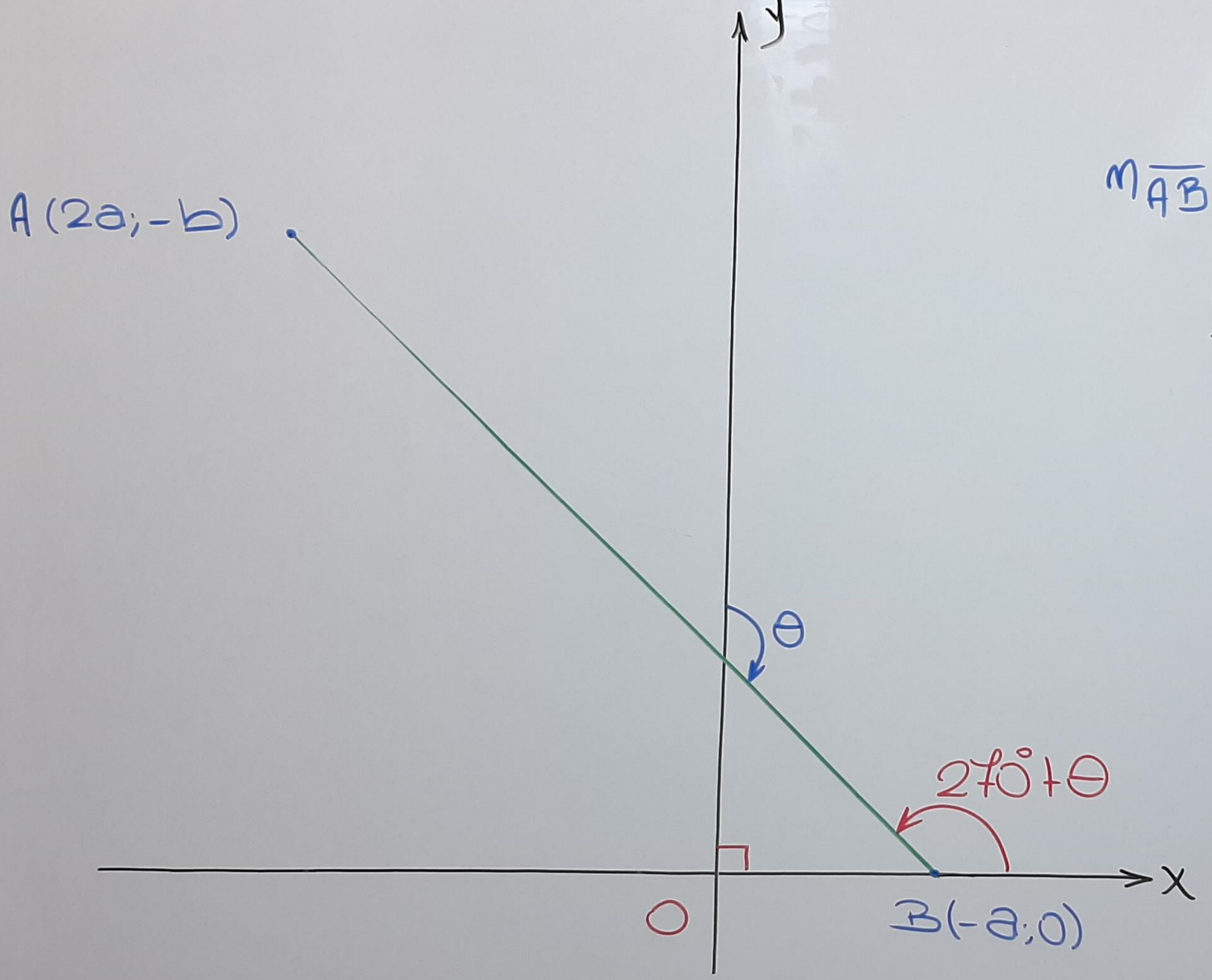
Problema 16:

Calcular $\tan\theta$.



A) $3a/b$
B) $b/3a$
C) $-b/3a$
D) $-3a/b$
E) $-a/3b$


y
(2a;-b)
θ
270°+θ
x
O
(-a;0)



$$m_{\overline{AB}} = \tan(\measuredangle inclin) = \frac{\Delta y}{\Delta x}$$

$$\tan(270° + \theta) = \frac{-b-0}{2a-(-a)}$$

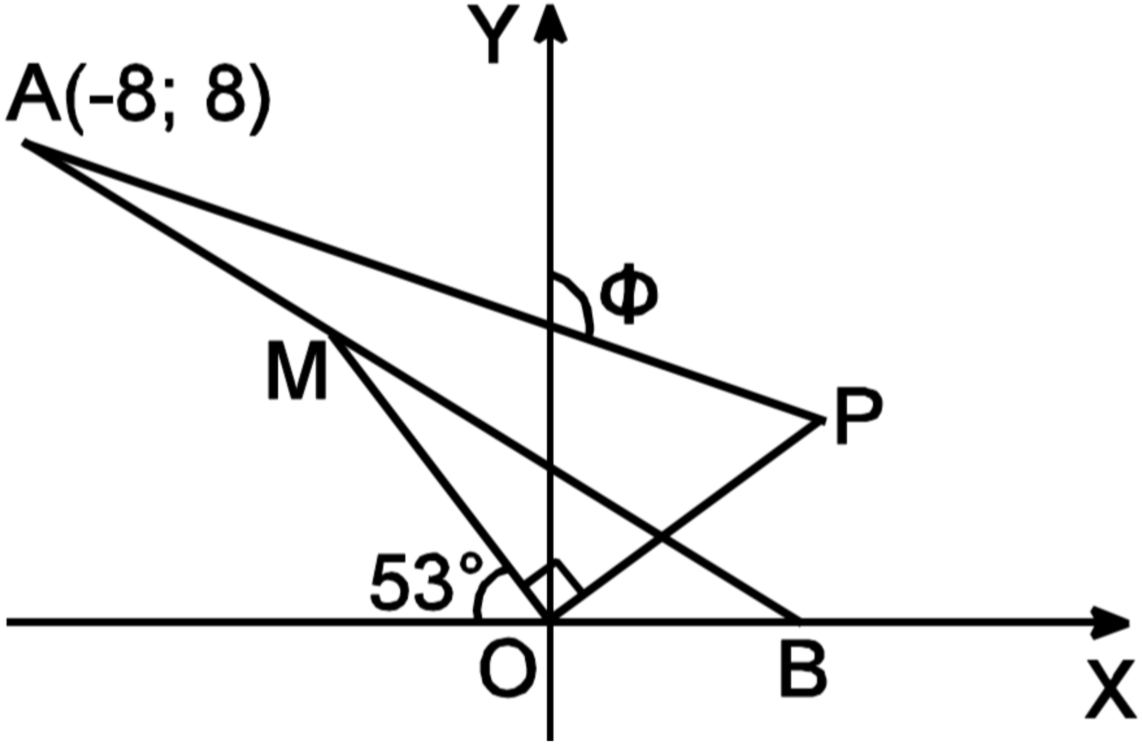
$$-\cot\theta = \frac{-b}{3a}$$

$$\therefore \tan\theta = \frac{3a}{b}$$

CLAVE A

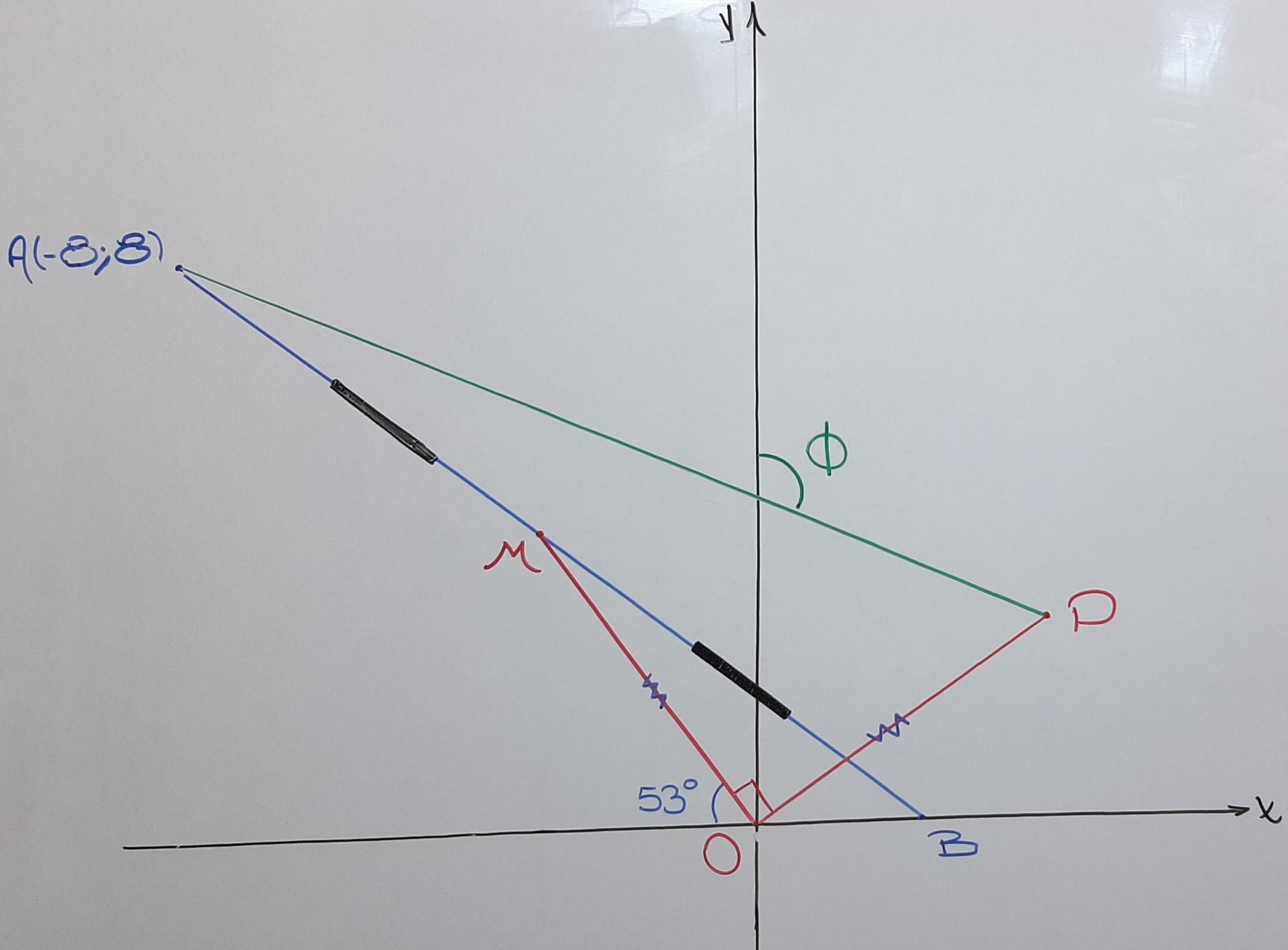
Problema 17:

A partir del grafico, calcule Cotφ, siendo M punto medio del segmento AB; MO=OP.

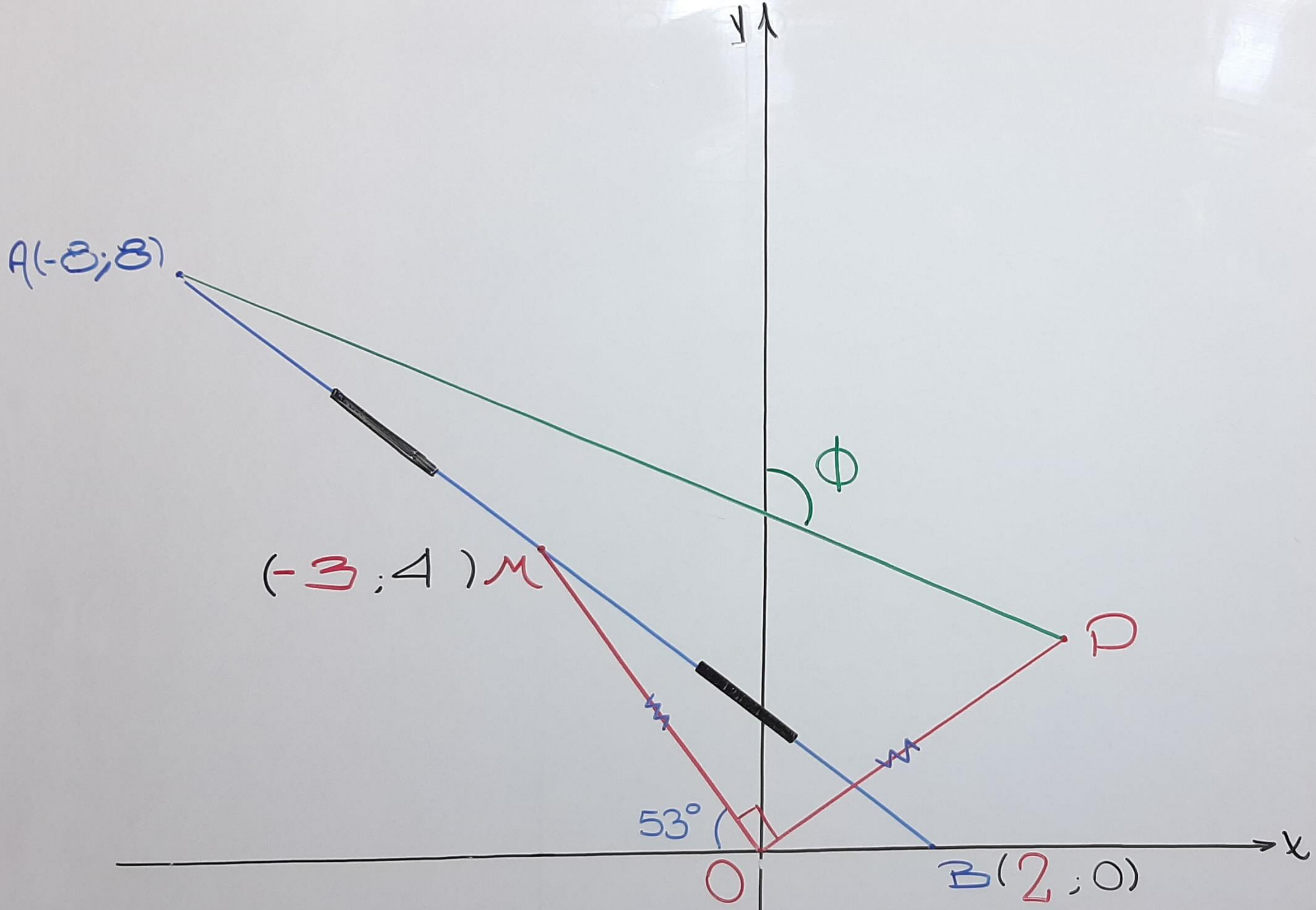


A) $-5/12$ B) $-12/5$ C) $5/12$

D) $1/2$ E) $1/3$

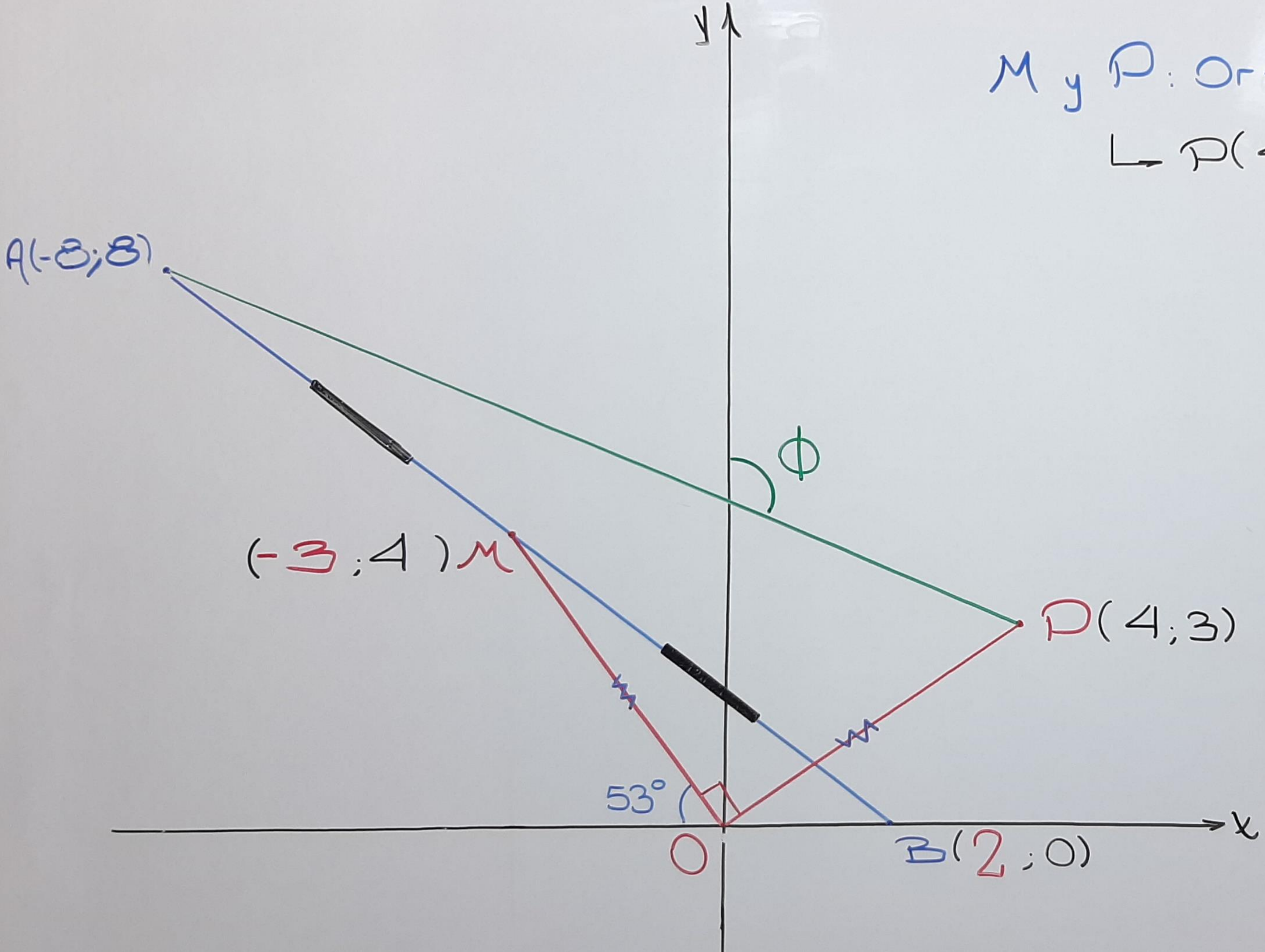


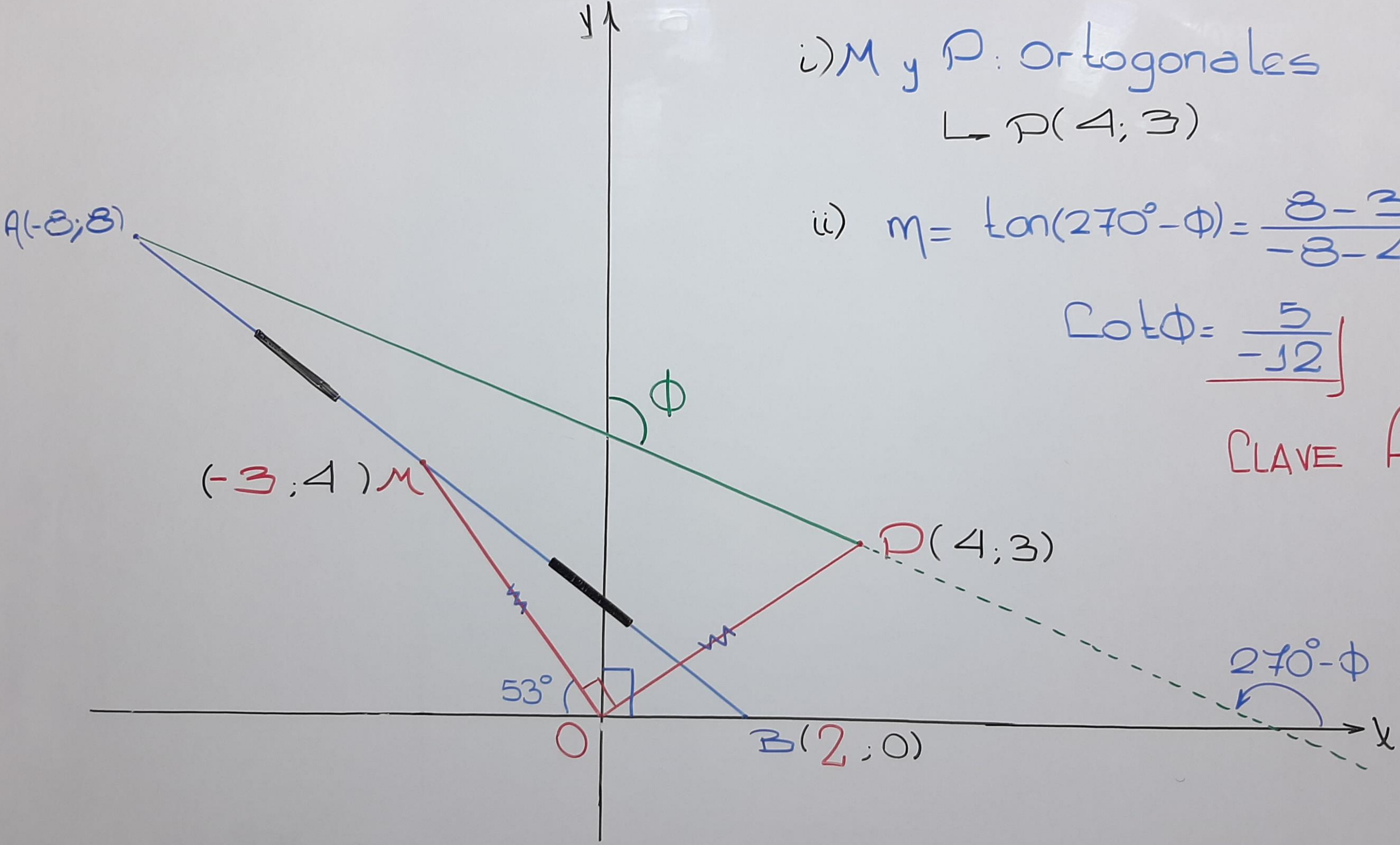
y
A(-8;8)
Φ
M
D
53°
x
O
B


y
x
A(-8;8)
(-3;4) M
D
ϕ
53°
O
B(2;0)

M y P: Ortogonales

↳ P(4; 3)





i) M y P: ortogonales

↳ P(4; 3)

ii) $m = \tan(270° - \Phi) = \dfrac{8-3}{-8-4}$

$$\boxed{\cot\Phi = \dfrac{5}{-12}}$$

CLAVE A

Problema 9 (tarea):

Si a; b y c $\in \mathbb{Z}$, simplificar:

$$\mathrm{Cos}\left[(2a + c + 1)\pi - (4b + 1)\frac{\pi}{2} + \alpha\right]$$

A) $(-1)^{a}\mathrm{Sen}\alpha$ B) $(-1)^{b}\mathrm{Sen}\alpha$ C) $(-1)^{c}\mathrm{Sen}\alpha$
D) $(-1)^{c+1}\mathrm{Sen}\alpha$ E) $(-1)^{a+b}\mathrm{Sen}\alpha$

a, b y $c \in \mathbb{Z}$

$$\cos\left[(2a+c+1)\pi - (4b+1)\frac{\pi}{2} + \alpha\right]$$

$$\cos\left[2a\pi + c\pi + \pi - 2b\pi - \frac{\pi}{2} + \alpha\right]$$

$a, b \text{ y } c \in \mathbb{Z}$

$$\cos\left[(2a+c+1)\pi - (4b+1)\frac{\pi}{2} + \alpha\right]$$

$$\cos\left[2a\pi + c\pi + \pi - 2b\pi - \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{2(a-b)}_{\text{Par}}\pi + c\pi + \frac{\pi}{2} + \alpha\right]$$

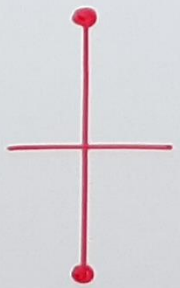
a, b y $c \in \mathbb{Z}$

$$\cos\left[(2a+c+1)\pi - (4b+1)\frac{\pi}{2} + \alpha\right]$$

$$\cos\left[2a\pi + c\pi + \pi - 2b\pi - \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{2(a-b)}_{\text{Par}}\pi + c\pi + \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{(2c+1)\frac{\pi}{2} + \alpha}\right]$$



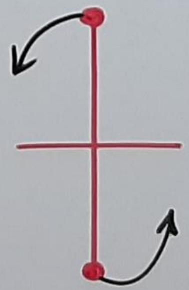
a, b y $c \in \mathbb{Z}$

$$\cos\left[(2a+c+1)\pi - (4b+1)\frac{\pi}{2} + \alpha\right]$$

$$\cos\left[2a\pi + c\pi + \pi - 2b\pi - \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{2(a-b)}_{\text{Par}}\pi + c\pi + \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{(2c+1)\frac{\pi}{2} + \alpha}\right] \longrightarrow$$



Si c: par $\rightarrow \measuredangle \in \overrightarrow{OY}$

$\Rightarrow -\operatorname{Sen}\alpha$

Si c: impar $\rightarrow \measuredangle \in \overrightarrow{OY'}$

$\Rightarrow +\operatorname{Sen}\alpha$

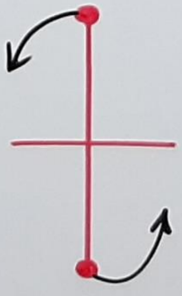
a, b y $c \in \mathbb{Z}$

$$\cos\left[(2a+c+1)\pi - (4b+1)\frac{\pi}{2} + \alpha\right]$$

$$\cos\left[2a\pi + c\pi + \pi - 2b\pi - \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{2(a-b)}_{\text{Par}}\pi + c\pi + \frac{\pi}{2} + \alpha\right]$$

$$\cos\left[\underbrace{(2c+1)\frac{\pi}{2} + \alpha}\right] \longrightarrow$$



Si c: par $\rightarrow \measuredangle \in \overrightarrow{OY}$

$\Rightarrow \; -\operatorname{Sen}\alpha$

Si c: impar $\rightarrow \measuredangle \in \overrightarrow{OY'}$

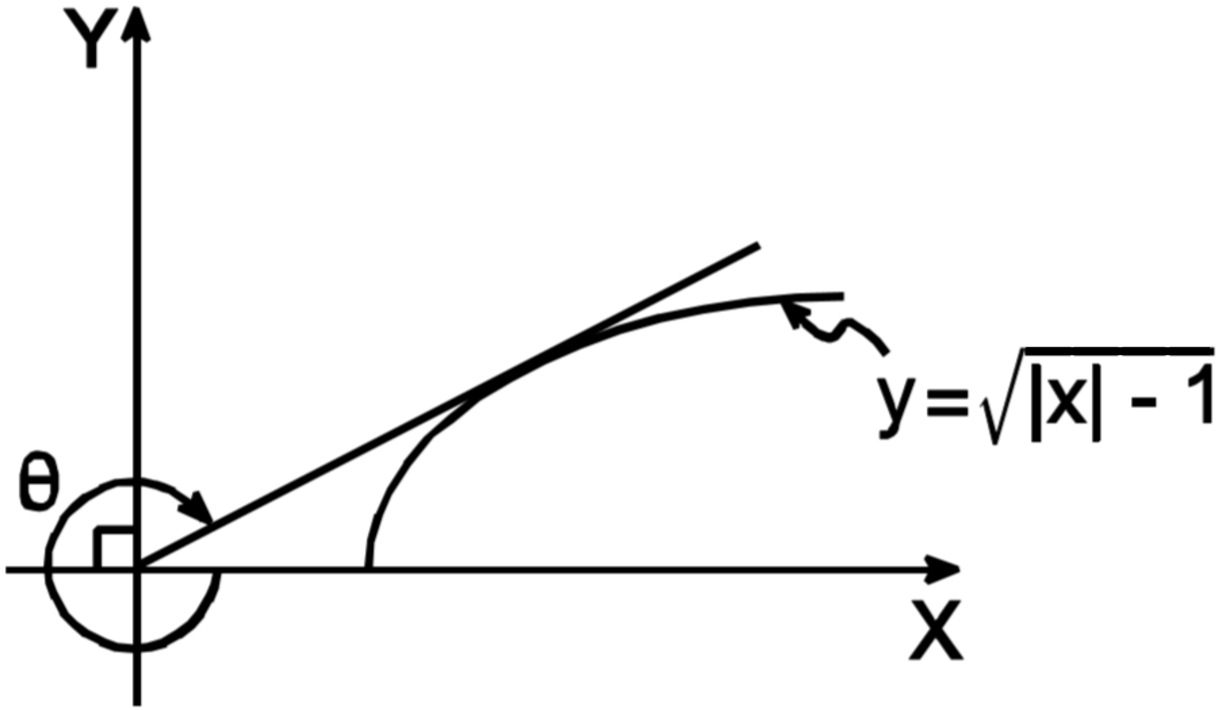
$\Rightarrow \; +\operatorname{Sen}\alpha$

$$(-1)^{c+1} \cdot \operatorname{Sen}\alpha$$

CLAVE D

Problema 19:

Del grafico mostrado, determine Tanθ.



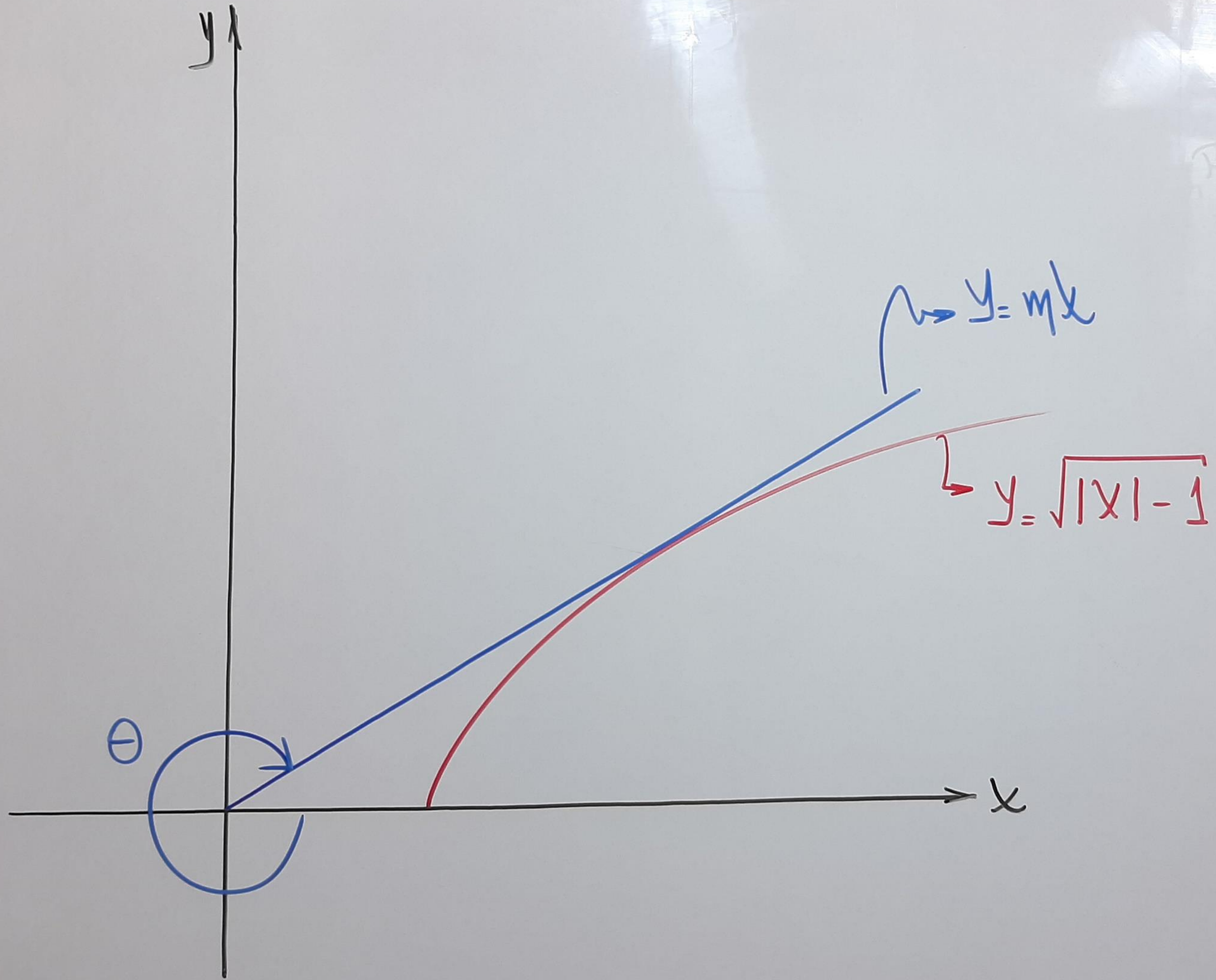
A) $1/2$

B) $\sqrt{2}$

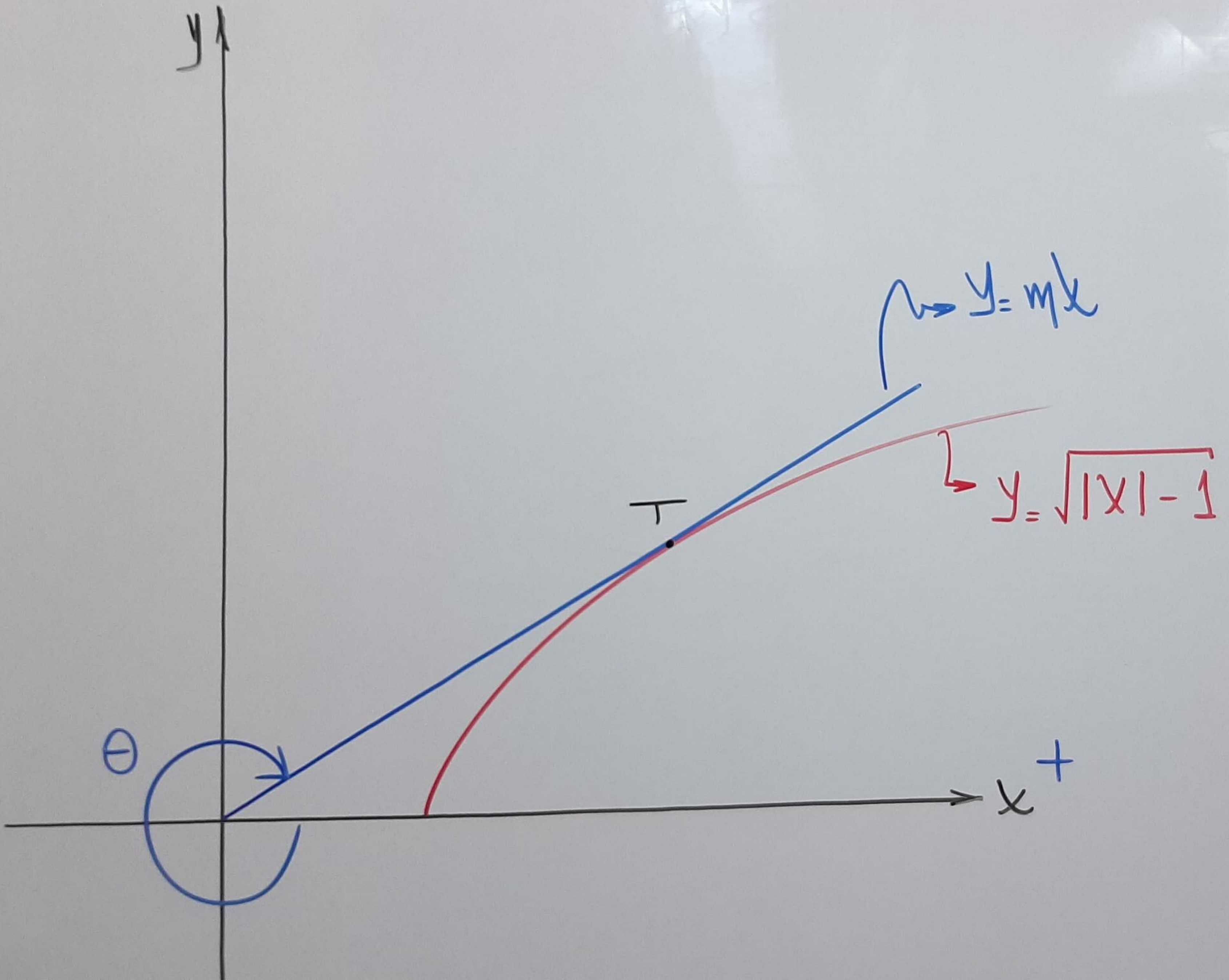
C) $\sqrt{2}/2$

D) $\sqrt{3}/2$

E) 1



y
x
θ
y = mx
y = √(|x| - 1)



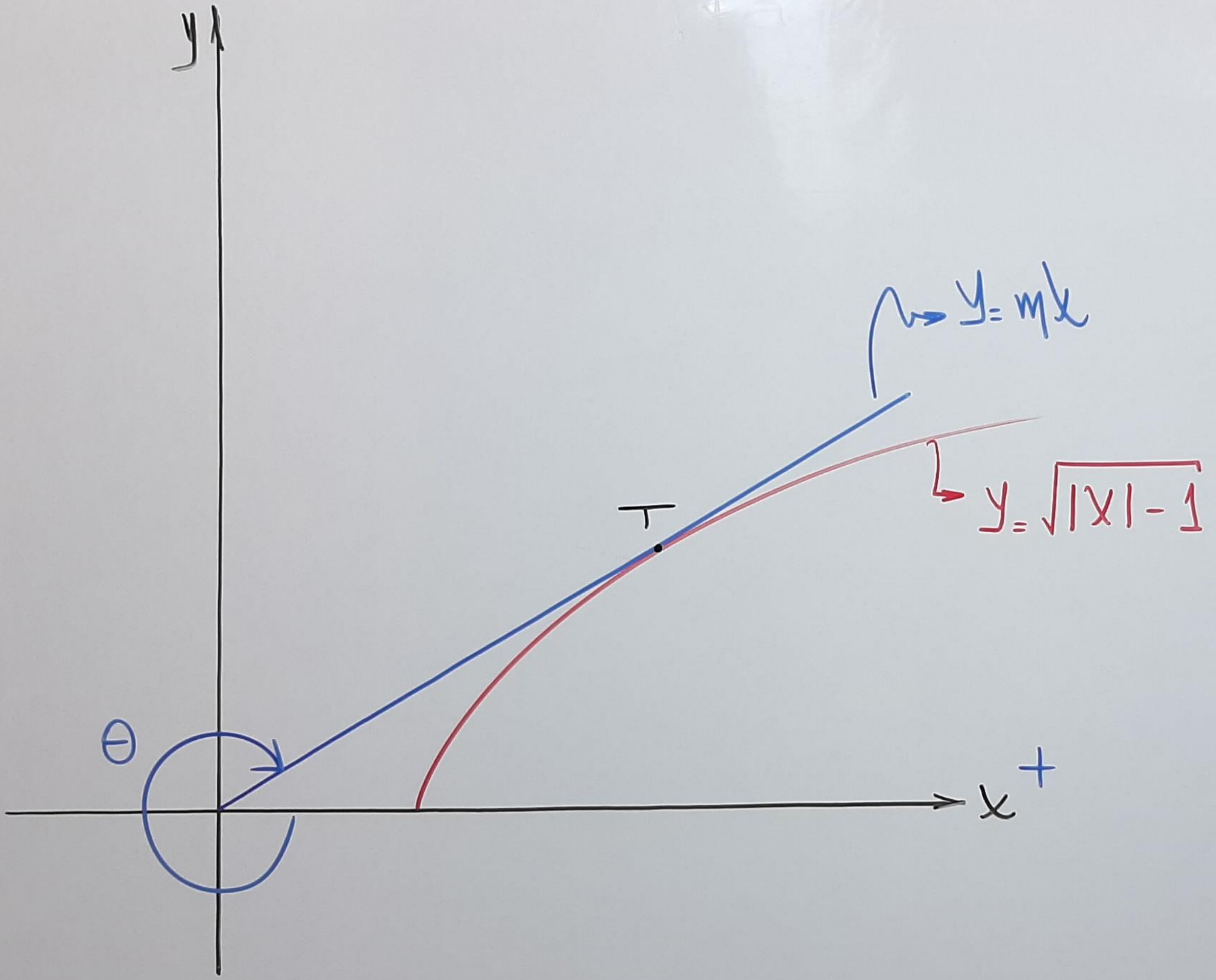
En T:

$$y = mx = \sqrt{|x| - 1}$$

$$m^2x^2 = |x| - 1$$

$$m^2x^2 = x - 1$$

$$m^2x^2 - x + 1 = 0$$



En T:

$$y = mx = \sqrt{|x| - 1}$$

$$m^2x^2 = |x| - 1$$

$$m^2x^2 = x - 1$$

$$m^2x^2 - x + 1 = 0$$

T (Pto de Tangencia)

$$\Delta = 0$$

$$(-1)^2 - 4(m^2)(1) = 0$$

$$m = \pm\frac{1}{2}$$

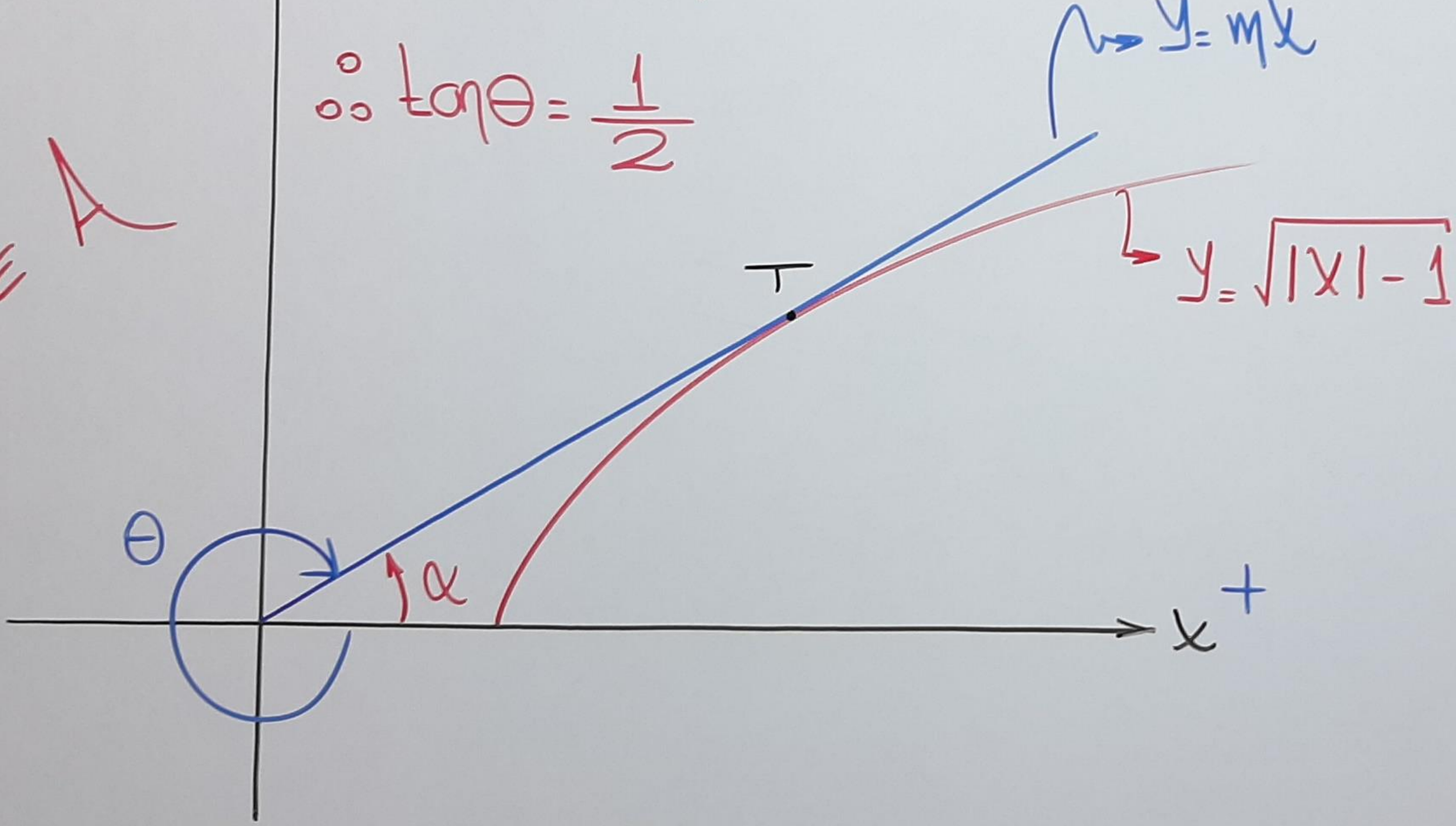
$$m = \frac{1}{2}$$

α y θ: ∡ Coterminales

$\tan\theta = \underbrace{\tan\alpha}_{}$

$\tan\theta = m$

$\therefore \tan\theta = \dfrac{1}{2}$



CLAVE A

En T:

$y = mx = \sqrt{|x|-1}$

$m^2x^2 = |x| - 1$

$m^2x^2 = x - 1$

$m^2x^2 - x + 1 = 0$

T (Pto de Tangencia)

$\Delta = 0$

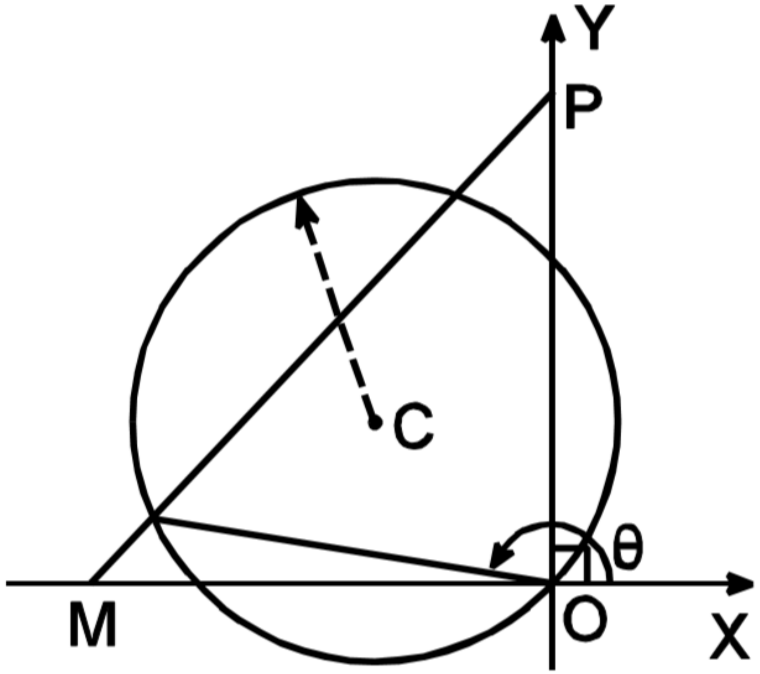
$(-1)^2 - 4(m^2)(1) = 0$

$m = \pm\dfrac{1}{2}$

$m = \dfrac{1}{2}$

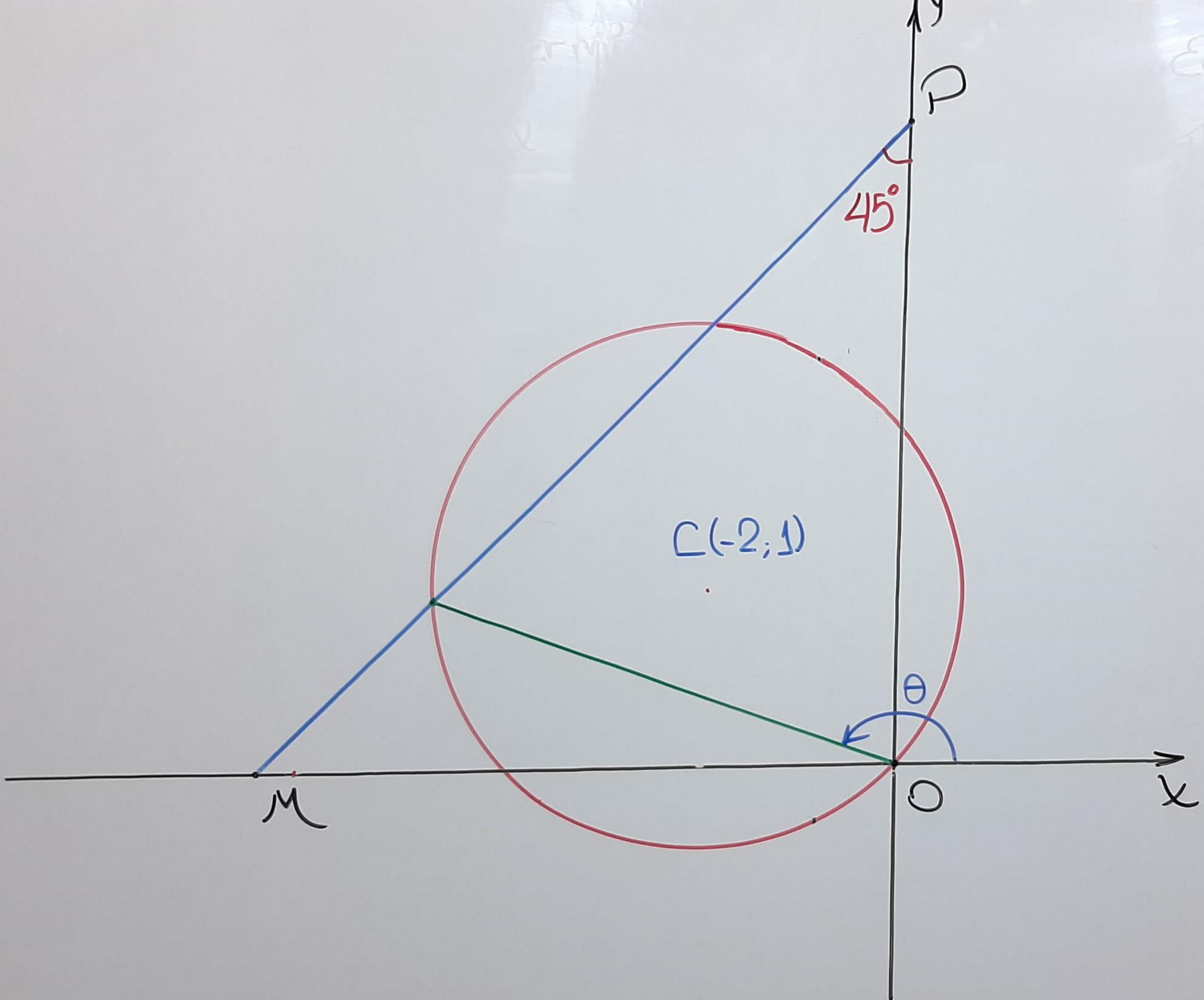
Problema 20:

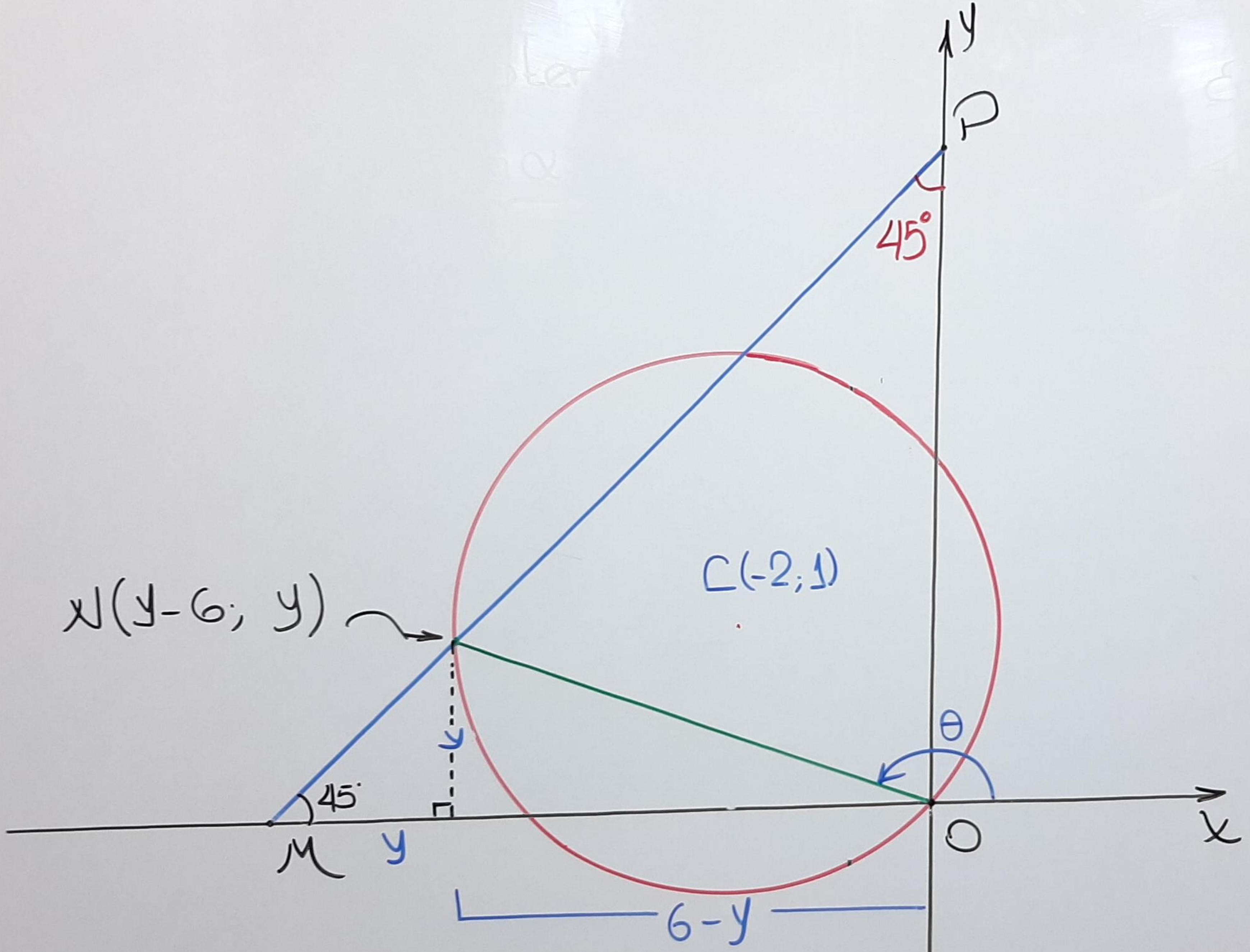
Del grafico mostrado: $C(-2;1)$ es centro. Obtener el valor de: $N = \text{Tan}\theta - \text{Sec}\theta$
Si PO=OM=6



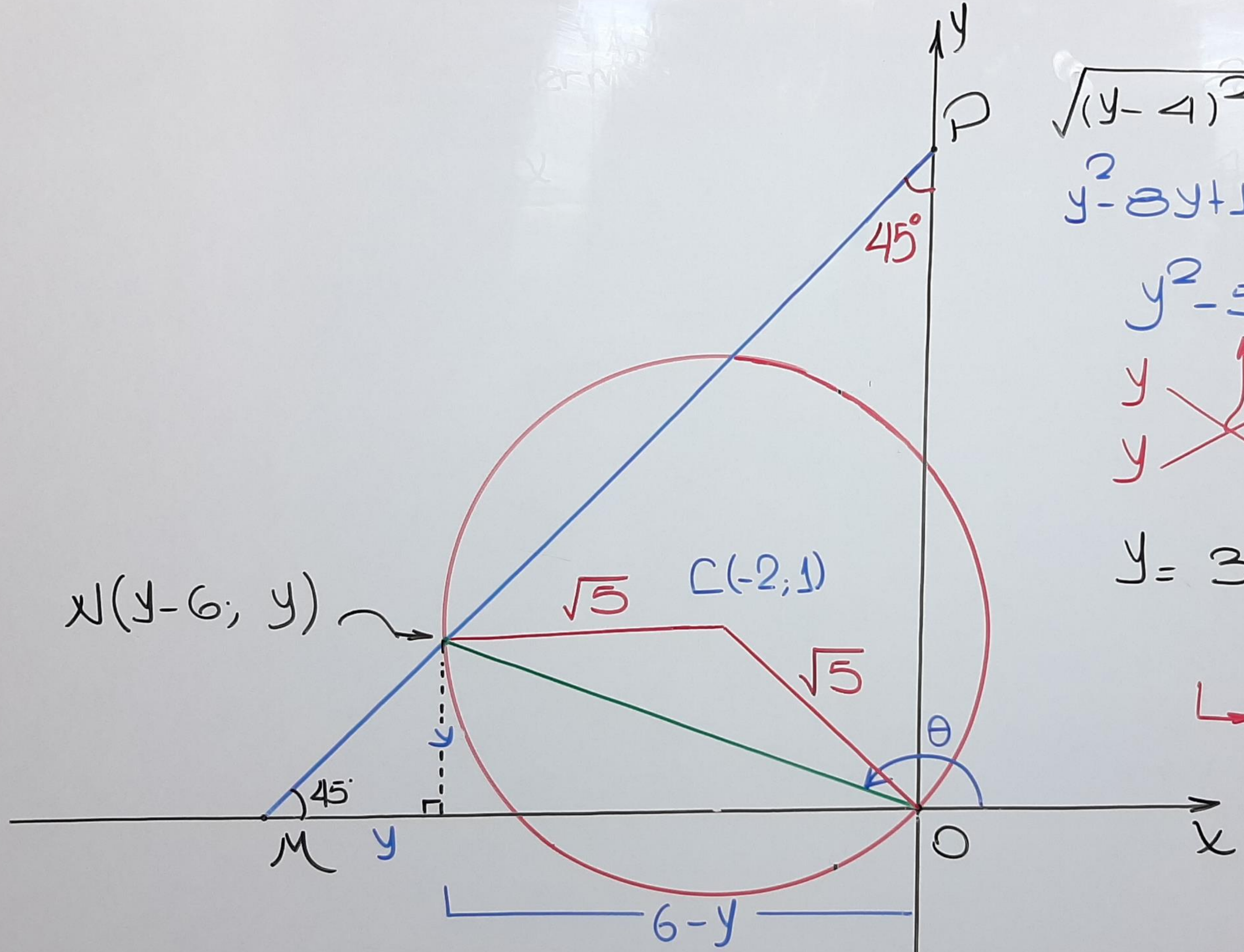
A) $2+\sqrt{5}$ B) $2-\sqrt{5}$ C) $\frac{\sqrt{5}+1}{2}$

D) $\frac{\sqrt{5}-1}{2}$ E) $\sqrt{5}-2$


y
P
45°
C(-2;1)
θ
M
O
x



y
P
45°
C(-2;1)
N(y-6; y)
θ
y
45°
M
y
O
x
6 - y



$$\sqrt{(y-4)^2+(y-1)^2}=\sqrt{5}$$

$$y^2-8y+16+y^2-2y+1=5$$

$$y^2-5y+6=0$$

$$\begin{array}{ll} y & -3 \\ y & -2 \end{array}$$

$$y=3 \vee y=2$$

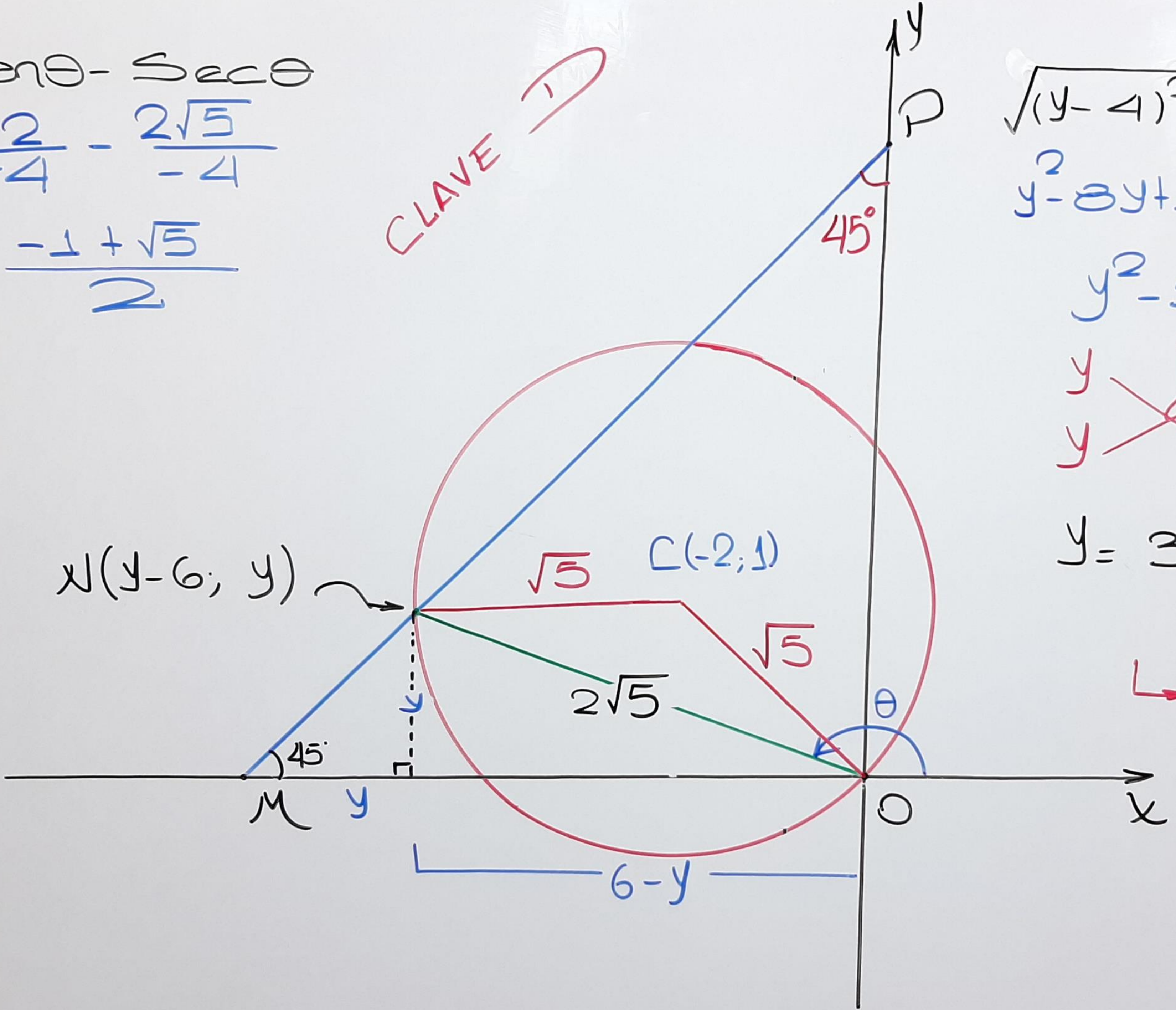
$$\hookrightarrow N(-4;2)$$

$\tan\theta - \sec\theta$

$\frac{2}{-4} - \frac{2\sqrt{5}}{-4}$

$\frac{-1+\sqrt{5}}{2}$

CLAVE



$\sqrt{(y-4)^2+(y-1)^2} = \sqrt{5}$

$y^2-8y+16+y^2-2y+1=5$

$y^2-5y+6=0$

$y \quad -3$

$y \quad -2$

$y=3 \lor y=2$

$\rightarrow N(-4;2)$

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS